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DUAL-CRITERIA METHOD FOR DETERMINING CRITICAL PLANE ORIENTATION FOR MULTIAXIAL FATIGUE PREDICTION USING A GENETIC ALGORITHM

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ABSTRACT

This paper is an attempt to formulate an estimation method which can efficiently determine the critical plane according to the criteria under consideration. It is required to maintain greater accuracy than the incremental angle methods used conventionally in critical plane searching algorithms. The multi-criteria-based critical plane selection method is evaluated; the considered criteria include a fatigue parameter and variance of shear stress, both maximized to find the most damaging plane. The results show that the proposed model reduces the number of iterations by 90% with greater accuracy than the conventional methods and multiple candidate planes can easily be identified. Two or more criteria can easily be implemented in searching for the critical plane. The GA-based critical plane location method shows promise for fatigue life estimation as it is flexible and simple to implement.

***Keywords*:** Critical plane; multiaxial fatigue; genetic algorithm; dual criteria.

INTRODUCTION

Fatigue failures have been investigated for over a century but still the scientific community has reached no agreement on the methodology appropriate for fatigue life analysis of mechanical components in service variable amplitude loading [1-6]. Multiaxial fatigue analysis is one of the major issues in fatigue modeling development and to formulate an accurate criterion which can handle these type of situations is a prime concern in ongoing research. To address the above issue researchers have proposed methodologies to tackle fatigue life assessment under time-variable multiaxial loading conditions. Many different methodologies have been proposed which are based on various initial concepts [7-13]. Among the various techniques proposed so far, the critical plane approach is essentially based on experimental observations that cracks initiate in preferential material planes, usually associated with high shear stresses [14, 15]. This approach propose that, at a crack initiation site a plane where maximum damage will occur is the one facing maximum shear stress amplitude [16-18].

In the present work a critical plane estimation method is proposed which is based on optimization techniques. The single fatigue parameter and dual parameter setups are investigated. In the single fatigue parameter study critical plane results are compared between plane angles calculated from incremental angle method and GA when using the fatigue parameter. The objective is to judge the efficiency and accuracy of GA compared with conventional angle increments. Two fatigue parameter setup to determine the critical plane is developed by taking advantage of multi-objective optimization in GA using MOGA II. The objective is to observe the effect of at least two dissimilar criteria which can help better estimation of the critical plane and can perform better than when only one criterion is used which can miss the critical plane on which maximum damage is occurring in real-world complex scenarios. A similar situation is identified by Araujo, Dantas [14] in which they identified the critical plane from many candidate planes with similar maximum shear stress amplitude (i.e. initial criteria of the fatigue model to identify critical plane) by selecting the critical plane with the help of an additional criteria of normal stress. In the current study the problem of more than one candidate plane determined according to a selected criterion is solved by using more than one fatigue parameter addressing different aspects of applied loading.

Methods and Materials

Fatigue Parameters

One of the parameters under consideration is based on a newly proposed model by the author as shown in Eq. (1) and the parameter for the critical plane is shown in Eq. (2). The new model is based on normal and shear strain and maximum normal and shear stress values on the critical plane. The coefficients are calibrated through experiential fatigue life results. The fatigue parameter (Eq. 2) is derived from the same model by assuming it as the straight sum of all stress-strain quantities considered in model equation (Eq. 1(a)), to obtain the plane with the maximum combined effect of the constituent quantities of the proposed model.

$$P = a_1 \cdot \Delta\gamma . + a_2 \cdot \left(\frac{\tau_{\max}}{\sigma_Y}\right) + a_3\,\Delta\varepsilon + a_4 \cdot \left(\frac{\sigma_{n,\max}}{\sigma_Y}\right) - \frac{{\sigma_L}^2}{E} - ev \tag{1a}$$

$$ev = V.dP^m \left(\frac{\tau_{\max} - \tau_m}{E}\right)\left(\frac{\sigma_{\max} - \sigma_m}{E}\right) \tag{1b}$$

$$D_n = K.P^R.\Delta P \tag{1c}$$

$$P = \Delta\gamma . + \left(\frac{\tau_{\max}}{\sigma_Y}\right) + \Delta\varepsilon + \left(\frac{\sigma_{n,\max}}{\sigma_Y}\right) \tag{2}$$

The other fatigue parameter used in the study is variance of shear maximized on the critical plane [19]. Equations involving the calculation of variance of shear were included in this study as Module 1 (as described in Susmel [19]) and Module 2 consisting of an optimization algorithm is left out, as in the current study a genetic algorithm is used to estimate the critical plane. The expression used to determine variance [19] is shown in Eq. (3).

$$\mathrm{Var}(\tau) = d^T\,[C]\,d \tag{3}$$

where d is the vector of direction cosines and C is matrix of variance and co-variance terms.

Analytical Modeling

Two sets of applied loading from the published literature with carbon steel C40 and stainless steel SS304 are considered in this study. Loading set for C40 [20] consists of in phase and out of phase tension and torsion loading with zero and positive mean with different magnitude of applied load. The load set for SS304 [21, 22] consist of various non-proportional loading paths. Stress and strain results from finite element analysis of the test specimens were used for determination of the critical plane. The loading profile for C40 is sinusoidal and loading paths for SS304 are shown in Table 1, with load values for C40 and SS304 cases. Dimensions of specimens used in the study are shown in Figure 1. In both cases the notch root is assumed to be the most damaging point used for critical plane determination. The material properties of C40 and SS304 are stated in Table 2. The angles theta (θ) and phi (ϕ) used to locate the critical plane are defined in Figure 2 [16] and Figure 3 shows the flow chart of the process used in this study, highlighting the incremental, single and two-parameter setups.

Table 1. Profile paths for SS 304 (ε – x axis and γ – y axis) [21].

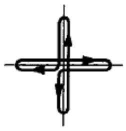
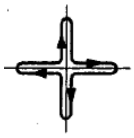
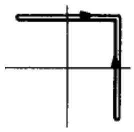
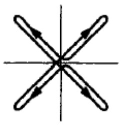
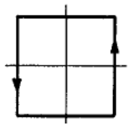
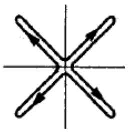
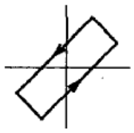
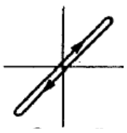
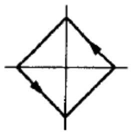
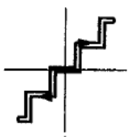
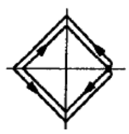
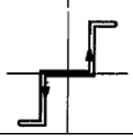
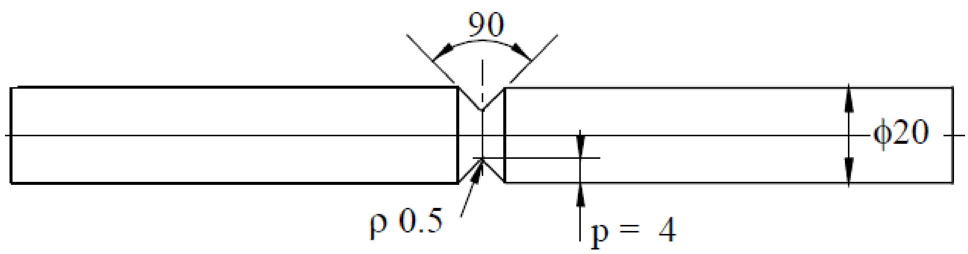
Loading Case No.	Path Shape	Loading Case No.	Path Shape
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7			

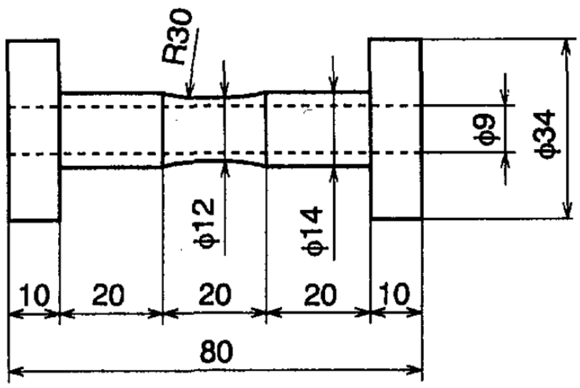
Table 2. Material properties of C49 and SS304.

Material Name	Young's modulus [23]	Yield stress (MPa)	Ultimate tensile strength (MPa)	Cyclic strain hardening exponent, n'	Cyclic strength coefficient, K' (MPa)
C40[a]	206	537	715	0.131[b]	915[b]
SS304	197[c]	240[b]	898[d]	0.276[c]	1754[c]

Source: (a) [20] (b) [SAE J1099 (AUG2002)24] (c) [25] (d) [26]



(a) C40 specimen [20].



(b) SS304 specimen [21].

Figure 1. Dimensions of specimens

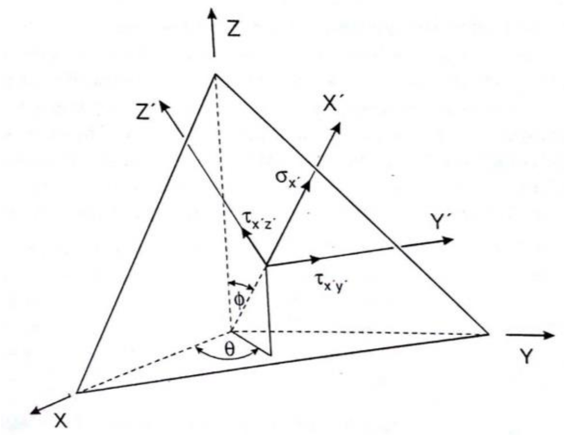


Figure 2. Theta (θ) and phi (ϕ) angles for locating critical plane [16].

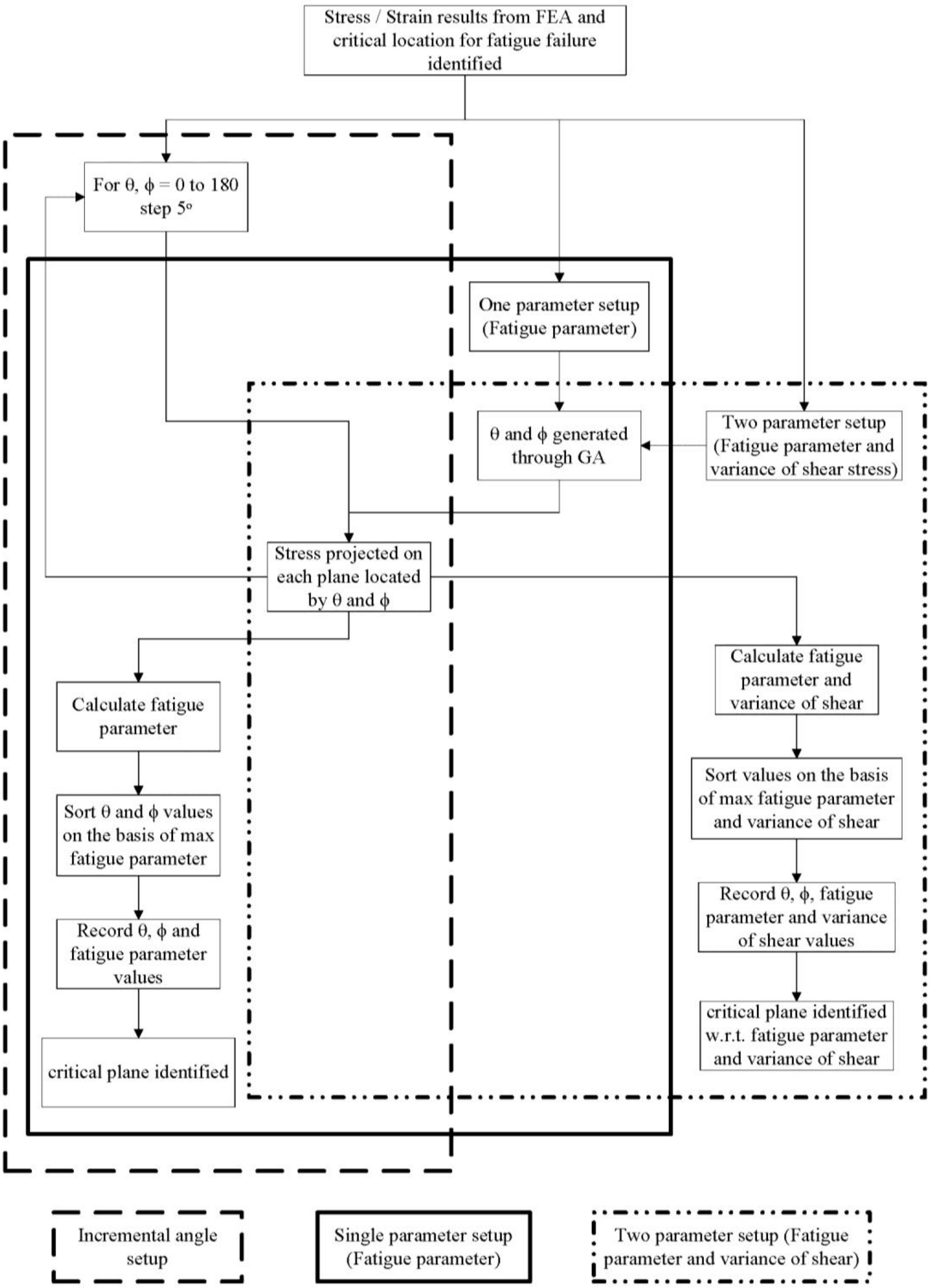


Figure 3. Process flow chart of critical plane estimation.

RESULTS AND DISCUSSION

In the current study a methodology has been proposed to estimate the critical plane for fatigue life determination. The proposed technique is based on genetic algorithm optimization and maximizing the fatigue parameter defined according to the fatigue criteria under consideration (Eq. 2). GA found the critical plane far more faster than the angle increment method, and also with greater accuracy, as GA applies decision-based generation of the value of critical plane angles to calculate the fatigue parameter; this in turn avoids the extra calculations on planes which have fatigue parameters of small value and reduces the effort needed to find a critical plane of the required accuracy. From the results in Tables 3 and 4, it can be seen that GA-based critical plane estimation has a least count of one degree and requires only around 1800 iterations (initial value set of 180 and number of generations 10) to locate the critical plane. For the incremental angle and the same least count of one degree we have to do more than 32000 iterations (i.e. 181 steps for each value of θ and φ angles) to get the fatigue parameter data to enable us to locate the critical plane for the maximized fatigue parameter. Figure 4(a) shows the fatigue parameter results from incremental angle setup for one of the applied loading cases and has many extra calculation points which are not needed for critical plane determination as the value of the fatigue parameter is very low for those values of θ and ϕ, this additional calculation is clearly seen to be reduced in the results of GA based fatigue parameter estimation as shown in Figure 4(b) for the same loading case, as those θ and ϕ causing low fatigue parameter values are ignored by GA. This is a huge improvement in performance, especially in terms of the time required for long complex multiaxial and variable amplitude loading. Also, it can be seen from some results in Tables 3 and 4 that the GA-based approach has located the critical plane with a fatigue parameter value higher than the one found by the incremental method as step size limitation means the incremental method cannot include that plane in its calculation and moreover at the same time GA has located more than one candidate for the critical plane. In some situations the results of incremental and GA-based methods are the same. This is because the loading profiles considered here have small variations which in turn do not create many candidate planes, so both methods result in the same orientation of the critical plane.

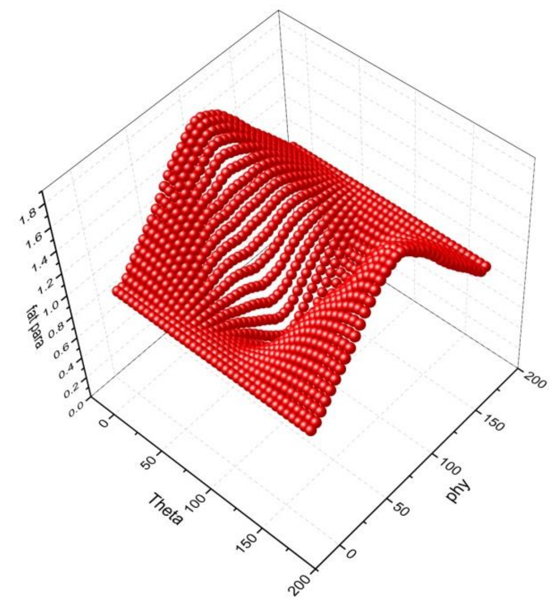
Table 3**.** Critical planes for C40 specimen.

Loading		Theta (θ)		Phi (ϕ)		Fatigue parameter	
Condition	Stress value (MPa)	Incremental	GA	Incremental	GA	Incremental	GA
R-1 Ph0	101	165	18 /165 /22	100	45 / 100 / 48	1.11054	1.11059 / 1.11057 / 1.11054
	200	180	178	145	143	1.41773	1.41980
R-1 Ph90	99.6	180	180	95	93	1.07621	1.07840
	199.7	0	179	90	88	1.81711	1.82106
R0 Ph0	67.9	170	164	140	138	1.32631	1.32647
	158.1	0	179 / 0	40	141/39	1.52013	1.52113 / 1.52085
R0 Ph90	66.8	180	0 / 180	95	85 / 97	1.45247	1.45248 / 1.4513
	158.1	0	179 / 0	95	85 / 95	1.88255	1.88317 / 1.88262

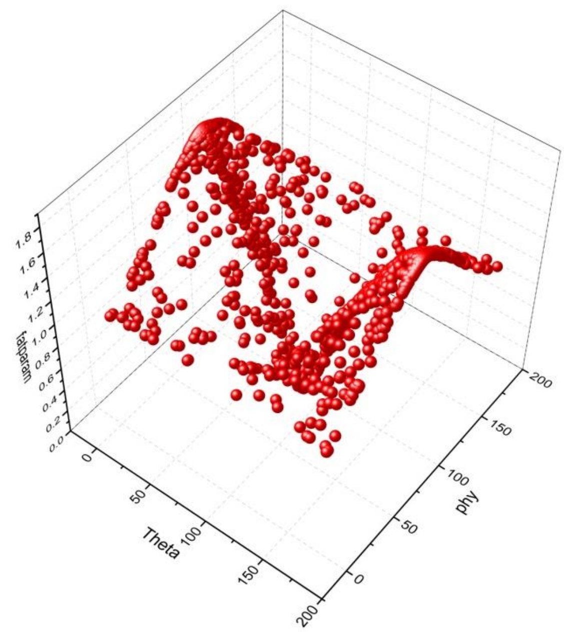
Table 4. Critical planes for SS304 specimen.

Case No.	Strain cases		Theta (θ)		Phi (ϕ)		Fatigue parameter	
	Axial	Shear	Incremental	GA	Incremental	GA	Incremental	GA
1	0.5	0.87	0	0	90	90	3.59584	3.59585
	0.8	1.39	0	0	90	90	4.33277	4.33277
2	0.5	0.87	0	180	90	91	3.58936	3.59028
	0.8	1.39	0	0	90	89	4.33162	4.33223
3	0.5	0.87	0	179 / 0	110	68 / 112	3.63267	3.64229 / 3.64147
	0.8	1.39	0	0	115	115	4.31325	4.31370
4	0.5	0.87	180	179	110	110	3.78136	3.78215
	0.8	1.39	180	179	110	111	4.45319	4.45605
5	0.5	0.87	175	0 / 176	35	144 / 37	2.91461	2.91645 / 2.91785
	0.8	1.39	175	174 /0	35	37 / 144	3.45054	3.45605 / 3.45206
6	0.5	0.87	180	178	80	80	3.308001	3.30986
	0.8	1.39	180	178	80	81	3.99819	4.004143
7	0.5	0.87	180	179	80	79	3.40371	3.40552
	0.8	1.39	180	179 / 0	80	80 / 100	4.09305	4.09326 / 4.09305
8	0.5	0.87	0	179 / 0	120	62 / 116	3.54503	3.54953 / 3.53698
	0.8	1.39	180	179 / 0	80	79 / 101	4.53368	4.53555 / 4.5346
9	0.5	0.87	0	179 / 0	110	70 / 109	3.5469	3.5481 / 3.54489
	0.8	1.39	180	179 / 0	80	78 / 103	4.46761	4.47809 / 4.47752
10	0.5	0.87	180	180 / 0	100	103 / 78	3.82634	3.83397 / 3.83366
	0.8	1.39	180	179 / 0	105	104 / 76	4.55532	4.55786 / 4.55744
11	0.5	0.87	180	179	80	80	3.093013	3.093647
	0.8	1.39	180	180 / 0	80	79 / 101	3.71974	3.72325 / 3.72312
12	0.5	0.87	180	180	80	82	3.34163	3.346288
	0.8	1.39	180	180 / 0	80	80 / 100	3.943375	3.94338 / 3.94319
13	0.5	0.87	180	180	80	82	3.36139	3.366025
	0.8	1.39	180	180 / 0	80	80 / 100	3.95408	3.95423 / 3.95423

Results for the other aim of the study, to exploit GA for multi-objective optimization to find the critical plane with respect to more than one fatigue parameter, are shown in Table 5. The second parameter is variance of shear stress on the candidate plane [27]; by definition this parameter is best suited for variable amplitude loading. For the simple cyclic loading cases considered in this study, usually the identified planes with single parameter setup (newly proposed fatigue parameter) and two-parameter setup (newly proposed fatigue parameter and maximum variance of shear stress) are the same or very close to each other. This fact was also highlighted by the author of MVM [19], except for case nos. 3, 4, 5, 8, 9, and 10 where maximized variance resulted in different planes. As seen in Table 5, however, the applicability of and effect of using two parameters are clear; more refinement of the critical plane estimation takes place and extra candidate planes are identified.



(a)



(b)

Figure 4. Fatigue parameter estimation (a) with incremental angle (b) with GA.

Table 5. Critical plane with fatigue parameter and variance of shear stress maximized in SS 304 specimen.

Case No.	Strain cases		Theta (θ)		Phi (ϕ)		Fatigue parameter / Variance	
	Axial	Shear	Fatigue parameter	Variance	Fatigue parameter	Variance	Fatigue parameter	Variance
1	0.5	0.87	180	180	90	92	3.59586 / 1916372	3.59172 / 1920103
2	0.5	0.87	0	0	89	89	3.59028 / 491969	3.59027 / 491971
3	0.5	0.87	179	0 / 0	68	94 / 3	3.64221 / 572544	(3.05496 / 741350) / (1.70573 / 741335)
4	0.5	0.87	0 / 180	0 / 0	69 / 111	88 / 178	(3.78169 / 573812) / (3.781601 / 571695)	(3.23537 / 755789) / (1.63726 / 755789)
5	0.5	0.87	0 / 180 / 0	0 / 0 / 180	144 / 36 / 99	167 / 77 / 13	(2.91645 / 1000228) / (2.91641 / 1016364) / (2.91494 / 1009935)	(2.28644 / 2057109) / (2.29998 / 2057075) / (2.27207 / 2056764)
6	0.5	0.87	178 / 0	0 / 180 / 0	80 / 100	78 / 11 / 169	(3.30974 / 1910576) / (3.30804 / 1928486)	(2.65714 / 3504013) / (2.15744 / 3502552) / (2.15618 / 3502398)
7	0.5	0.87	0 / 180	180 / 0	100 / 80	100 / 80	(3.40408 / 1014914) / (3.40399 / 1024516)	(2.9106679 / 1615255) / (2.91041 / 1615246)
8	0.5	0.87	180/ 0	0 / 180	62 / 119	84 / 96	(3.54935 / 301002) / (3.54915 / 291789)	(2.71653 / 1237445) / (2.71330 / 1237415)
9	0.5	0.87	179 / 0	0 / 180 / 0	71 / 110	97 / 83 / 6	(3.54776 / 523944) / (3.54579 / 519767)	(3.29491 / 570816) / (3.30665 / 570795) / (1.91929 / 570464)
10	0.5	0.87	180 / 0	0 / 180 / 180	102 / 77	96 / 84 / 174	(3.83389 / 877440) / (3.83374 / 869146)	(3.38469 / 1196083) / (3.38125 / 1196032) / (1.77483 / 1196027)
11	0.5	0.87	180 / 0	0 / 180	80 / 100	77 / 103	(3.09337 / 238782) / (3.09301 / 245077)	(2.41156 / 492025) / (2.44128 / 491816)
12	0.5	0.87	0 / 180	180 / 0	98 / 82	96 / 84	(3.34628 / 499757) / (3.34628 / 499698)	(3.07728 / 582456) / (3.07864 / 582455)
13	0.5	0.87	180 / 0	0 / 180 / 180	82 / 98	89 / 91 / 0	(3.36598 / 869464) / (3.36597 / 867814)	(3.29182 / 927009) / (3.28955 / 926999) / (1.65672 / 926953)

CONCLUSIONS

A new methodology using the optimization algorithm has been proposed. A single parameter and dual parameter setups have been evaluated using a newly proposed fatigue parameter and a maximum variance of shear stress parameter. A comparison has been made between the proposed methodology and the conventional method of angle increments to locate the critical plane by maximizing the fatigue parameter. The results show an advantage of the proposed GA-based method over the incremental angle in terms of the number of iterations required. The multi-objective optimization feature of GA has been applied to maximize more than one fatigue parameter to locate the critical plane. Within the limitations of applied loadings, the results show the benefits of using two fatigue parameters and the results are refined after two-parameter critical plane estimation. A detailed study is needed with variable amplitude loadings and fatigue parameter combinations to test the findings. The proposed methodology can be exploited to the maximum for critical plane estimation in a variety of loading conditions in real-world service loadings.

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REFERENCES

[1] Papuga J. A survey on evaluating the fatigue limit under multiaxial loading. International Journal of Fatigue. 2011;33:153-65.

[2] Susmel L, Taylor D. A critical distance/plane method to estimate finite life of notched components under variable amplitude uniaxial/multiaxial fatigue loading. International Journal of Fatigue. 2012;38:7-24.

[3] Kamal M, Rahman MM, Sani MSM. Application of multibody simulation for fatigue life estimation. International Journal of Automotive and Mechanical Engineering. 2013;7:912-23.

[4] Kamal M, Rahman MM. Fatigue life estimation models: A state of art. International Journal of Automotive and Mechanical Engineering. 2014;9:1599-608.

[5] Kamal M, Rahman MM. Finite Element-Based Fatigue Behaviour of Springs in Automobile Suspension. International Journal of Automotive and Mechanical Engineering. 2014;10:1910-9.

[6] Kamal M, Rahman MM, Rahman AG. Fatigue life evaluation of suspension knuckle using multibody simulation technique. Journal of Mechanical Engineering and Sciences. 2012;3:291–300.

[7] Manson SS, Halford GR. Fatigue and durability of structural materials. Materials Park, Ohio: ASM International, Materials; 2006.

[8] Rahman MM, Kadirgama K, Noor MM, Rejab MRM, Kesulai SA. Fatigue life prediction of lower suspension arm using strain-life approach. European Journal of Scientific Research. 2009;30:437-50.

[9] Brighenti R, Carpinteri A. A notch multiaxial fatigue approach based on damage mechanics. International Journal of Fatigue. 2012;39:122-33.

[10] Susmel L, Taylor D. The Modified Wohler Curve Method applied along with the Theory of Critical Distances to estimate finite life of notched components subjected to

complex multiaxial loading paths. Fatigue and Fracture of Engineering Materials and Structures. 2008;31:1047-64.
[11] Klemenc J, Fajdiga M. Joint estimation of E–N curves and their scatter using evolutionary algorithms. International Journal of Fatigue. 2013;56:42-53.
[12] Saintier N, Palin-luc T, Bénabes J, Cocheteux F. Non-local energy based fatigue life calculation method under multiaxial variable amplitude loadings. International Journal of Fatigue. 2013;54:68-83.
[13] Susmel L, Tovo R, Socie DF. Estimating the orientation of Stage I crack paths through the direction of maximum variance of the resolved shear stress. International Journal of Fatigue. 2014;58:94-101.
[14] Araujo JA, Dantas AP, Castro FC, Mamiya EN, Ferreira JLA. On the characterization of the critical plane with a simple and fast alternative measure of the shear stress amplitude in multiaxial fatigue. International Journal of Fatigue. 2011;33:1092–100.
[15] Stephens RI, Fatemi A, Stephens RR, Fuchs HO. Metal fatigue in engineering. New York: John Wiley and Sons, Inc.; 2000.
[16] Socie DF, Marquis GB. Multiaixal Fatigue. Warrendale, PA, USA: Society of Automotive Engineers; 2000.
[17] Susmel L. Multiaxial notch fatigue. Abington Hall, Granta Park, Great Abington, Cambridge: Woodhead Publishing Limited; 2009.
[18] Ali N, Mustapa MS, Ghazali MI, Sujitno T, Ridha M. Fatigue life prediction of commercially pure titanium after nitrogen ion implantation. International Journal of Automotive and Mechanical Engineering. 2013;7:1005-13.
[19] Susmel L. A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems. International Journal of Fatigue. 2010;32:1875-83.
[20] Atzori B, Berto F, Lazzarin P, Quaresimin M. Multi-axial fatigue behaviour of a severely notched carbon steel. International Journal of Fatigue. 2006;28:485–93.
[21] Itoh T, Ohnami M, Sakane M, Socie DF. Nonproportional low cycle fatigue criterion for type 304 stainless steel. Journal of Engineering Materials and Technology. 1995;117:285-92.
[22] Kida S, Itoh T, Sakane M, Ohnami M, Socie DF. Dislocation structure and non-proportional hardening of type 304 stainless steel. Fatigue & Fracture of Engineering Materials & Structures. 1997;20:1375-86.
[23] Eitssayeam S, Intatha U, Rujijanagul G, Pengpat K, Tunkasiri T. Structural and electrical properties characterization of (1–x) PbZr0. 52Ti0. 48O3–xBaFe0. 5Nb0. 5O3 system. Applied Physics A. 2006;83:295-9.
[24] Technical report on low cycle fatigue properties ferrous and non-ferrous materials. USA: Society of Automotive Engineers; 2002.
[25] Meggiolaro MA, de Castro JTP. An improved multiaxial rainflow algorithm for non-proportional stress or strain histories – Part I: Enclosing surface methods. International Journal of Fatigue. 2012;42:217-26.
[26] Narayanasamy R, Loganathan C. The influence of friction on the prediction of wrinkling of prestrained blanks when drawing through a conical die. Materials and Design. 2007;28:904-12.
[27] Susmel L. A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems. International Journal of Fatigue. 2010;32:1875-83.