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VALIDATION OF CRACK INTERACTION LIMIT MODEL FOR PARALLEL EDGE CRACKS USING TWO-DIMENSIONAL FINITE ELEMENT ANALYSIS

R. Daud, A.K. Ariffin and S. Abdullah

Department of Mechanical and Materials Engineering, Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, 43600, Bangi, Selangor, Malaysia
Email: ruslizam@unimap.edu.my
Phone: +6049885323; Fax: +6049885167

ABSTRACT

Shielding interaction effects of two parallel edge cracks in finite thickness plates subjected to remote tension load is analyzed using a developed finite element analysis program. In the present study, the crack interaction limit is evaluated based on the fitness of service (FFS) code, and focus is given to the weak crack interaction region as the crack interval exceeds the length of cracks ($b > a$). Crack interaction factors are evaluated based on stress intensity factors (SIFs) for Mode I SIFs using a displacement extrapolation technique. Parametric studies involved a wide range of crack-to-width ($0.05 \leq a/W \leq 0.5$) and crack interval ratios ($b/a > 1$). For validation, crack interaction factors are compared with single edge crack SIFs as a state of zero interaction. Within the considered range of parameters, the proposed numerical evaluation used to predict the crack interaction factor reduces the error of existing analytical solution from 1.92% to 0.97% at higher a/W. In reference to FFS codes, the small discrepancy in the prediction of the crack interaction factor validates the reliability of the numerical model to predict crack interaction limits under shielding interaction effects. In conclusion, the numerical model gave a successful prediction in estimating the crack interaction limit, which can be used as a reference for the shielding orientation of other cracks.

***Keywords*:** Crack interaction limit; interacting cracks; shielding effects; fitness for service.

INTRODUCTION

The presence of an edge cracking strip in aging critical engineering structures, e.g., aerospace structures and pressure vessel components, always places the structural integrity at risk of failure. In structural safety assessments, the presence of isolated single edge cracks may be easier to conduct. For multiple surface cracks, the damage tolerance of a single crack is no longer applicable owing to the existence of crack interaction behavior between the cracks (i.e., shielding and amplification). The pattern of evolution of multiple crack interaction is different and becomes increasingly complex when two or more cracks are present in close proximity. Table 1 shows the past and most recent solution models for evaluating stress-shielding parameters in parallel cracks. The list of analytical models and approaches in Table 1 that are used to develop the model, can be categorized into energy release rate criterion, stress intensity criterion, stress field traction, and the combination of many approaches. These models have contributed to the advancement of stress shielding assessment.

Table 1. Analytical elastic shielding interaction models.

Parallel multiple cracks	Models and approaches used	References
Parallel and collinear cracks	Simultaneous Singular Integral based on Disturbance Stress	(Ratwani and Gupta, 1974)
Parallel-Collinear cracks	Coefficient of Interaction	(Yokobori et al., 1979)
Parallel cracks	Asymptotic Approximation	(Chang, 1982)
Row of periodic cracks	Fredholm Integral Equations	(Chen, 1987)
Parallel and collinear cracks	Kachanov Method by Superposition-Average Traction	(Kachanov, 1987, 1993, 2003; Kachanov and Laures, 1988; Kachanov and Montagut, 1986; Kachanov et al., 1990)
Offset parallel cracks	Edge function methods	(Dwyer, 1997)
Periodic array cracks	Hypersingular Integral Equation	(Choi, 1997)
Shedding arrays of edge cracks	Energy release rate	(Parker, 1999)
Micro-cracks	Actual Displacement Discontinuity	(Gorbatikh and Kachanov, 2000)
Collinear and parallel interface cracks	Mushkelishvili and Burger's vector	(Han et al., 2002)
Distributed parallel cracks	Rotated Staggered Grid	(Orlowsky et al., 2003)
Parallel cracks and collinear cracks	Modified Kachanov Method	(Gorbatikh et al., 2007; Li et al., 2008)
Inclined and offset parallel and collinear cracks	Integral Equation Method	(Chen, 2007; Saha and Ganguly, 2005)
Offset parallel cracks	Complex Stress Function Method	(Sankar and Lesser, 2006)
Parallel cracks	Muskhelishvili and Laurent's series	(Li et al., 2008)
Parallel symmetric cracks	Schmidt Method	(Yang, 2009)
Parallel surface cracks	Singular Integral Method	(Feng and Jin, 2009)
Periodic edge cracks	Fourier Integral	(Yildirim et al., 2011)
Parallel edge cracks	Fourier and Cauchy Integral	(Chen and Wang, 2012)

Safety assessment of the presence of multiple cracks is commonly referred to recommended FFS codes. However, the theoretical and practical examination of the recommended FFS codes, such as ASME Boiler and Pressure Vessel Code Section XI (ASME, 1998, 2004), API 579 (ASME, 2007), British Standard PD6495 (BSI, 1991) and BS7910 (BSI, 1997, 2005), Nuclear Electric CEGB R6 (R6, 2006), and JSME Fitness-for-Service Code (JSME, 2000), have found that the exception of crack interaction in FFS combination rules has resulted in over-estimated and unrealistic fracture and failure predictions, particularly for closely spaced cracks. Burdekin (1982) presented elastic and plastic crack interaction solutions based on crack open displacement and the *J*-integral.

The formulation by Iida (1983) established a new single coalesced crack, but crack interaction is excluded. Moreover, based on the Iida solution, Leek and Howard (1994a) proposed an estimation of a single coalesced crack and a function of crack interaction factor (Leek and Howard, 1994b, 1996) corresponding to stress intensity factors (SIFs) at crack tips obtained by finite element (FE) analysis. An FE alternating method has been developed by O'donoghue et al. (1984) to solve the problem of interacting multiple cracks in a finite solid. Moussa et al. (1999) used the finite element method (FEM) to calculate the J-integral and used it to analyze the interaction of two identical parallel non-coplanar surface cracks subjected to remote tension and pure bending loads in a 3D finite body. The application of the body force method (BFM) by Kamaya (2003) justified the direction of coalesced cracks with a proposed new formulation. Owing to the limitations of BFM, FEM and the virtual crack extension method is employed by Kamaya (2008a, 2008b) to investigate the formation of a single coalesced crack as a result of crack growth under fatigue loading. In creep loading conditions, Xuan et al. (2009) introduced a creep interaction factor to address the interaction between cracks as a reaction to the stress field under loading. Kamaya et al. (2010) used an S-version FEM to analyze the crack growth of surface cracks. In relation to the FFS code limitations, crack interaction problems are more concentrated on elastic crack interaction with crack propagation, rather than on elastic crack interaction without crack propagation. In the past, many techniques have focused on the strong interaction region, particularly on the combination rule of parallel to coplanar (shielding effect), and the assumption of two cracks as a single crack for coplanar cracks (amplification effect). Most proposed solutions have considered mainly the strong interaction region and none of them question how crack interaction will behave within the weak interaction region, especially when the interaction approaches the crack interaction limit (CIL) (Zulkifli et al., 2011). To date, there are few investigations with emphasis on investigating the shielding effects in the weak interaction region as the interaction approaches CIL.

This paper presents a numerical approach for modeling multiple edge strip cracks that experience weak elastic crack interaction in a finite plate (Kamal et al., 2012; Domínguez Almaraz et al., 2010). The crack interaction is limited to crack interaction without crack propagation, where the shielding effect is dominated in promoting fracture and failure. The aim of this paper is to study the effect of the relative position of two parallel edge cracks, subjected to a variation of crack interval b/a and crack-to-width ratio a/W. The numerical analysis was performed using 2D linear FE analysis using developed APDL codes in ANSYS. The obtained value of the SIFs and elastic crack interaction factor are compared with corresponding FFS rules and numerical data from literature.

For the present investigation of elastic CIL, the BSI codes (1991, 1997, 2005) statements and the works of Jiang et al. (1990, 1991, 1992), and Leek and Howard (1994a, 1994b, 1996) are taken as the interaction limit reference. It is used for determining the onset of weak interaction and where the weak interaction is diminished, focusing specifically on the effect of shielding for parallel edge cracks in a finite body. For reference, the elastic crack interaction (ECI) of all parallel cracks may be redefined to diminish approximately at $b \geq 3a$ with the assumption that the crack lengths

$a_i (i = 1, 2....n)$ are equal length. The new analytical ECI factor γ_{ECI} to address the CIL problem is proposed and it written as Eq. (1)

$$\gamma_{ECI} = \gamma_{ECI \to CIL}(a_i / W, b / a_i) \quad ,(i = 0,1,2,...n) \tag{1}$$

and needs to comply with FFS codes in the condition of

$$\begin{aligned} &\gamma_{ECI}(a_i / W, b / a_i) \propto FFS \text{ (ASME,BSI,JSME)} \\ &\gamma_{ECI}(a_i / W, b / a_i) \propto FFS \ (b \geq 12.7\text{mm}, b \geq (a_1 + a_2)/2, b > 10\text{mm}) \end{aligned} \tag{2}$$

CRACK INTERACTION LIMIT DETERMINATION

The new model of CIL is based on the formulation of the state of no interaction or a single cracked body to the state of having weak interaction, until the interaction becomes stronger as the crack interval decreases. The elastic CIL is based on the analytical works of Jiang et al. (1990, 1991, 1992), and Leek and Howard (1994a, 1994b, 1996) as reference. The SIF calculation is limited to a linear elastic problem with a homogeneous, isotropic material near the crack region. The SIF determination is based on the creation of a singular element at the crack tip based on a quadratic isoparametric finite element. A quarter-point singular element or 8-node collapsed quadrilateral element developed by Henshell and Shaw (1975) is used; this has distinct advantages in terms of time, meshing, and re-meshing over the other quarter-point element (Banks Sills, 2010). The singularity is obtained by shifting the mid-side node ¼ points close to the crack tip. To calculate the SIF, we assumed the elements to be in rigid body motion and constant strain modes. The accuracy of this special element has been addressed by Murakami (1976), where the crack tip nodal point is enclosed by a number of special elements and in analysis, the size, number, and compatibility of special elements really affect the accuracy.

By assuming the crack interaction will be diminished at $b \geq 3a_i$ and therefore, it is about equivalent to the SIF of a single crack in the state of zero crack interaction factor, the analytical single crack Mode I SIF proposed by Brown and Strawley (1966) is used as the SIF reference $K_{I\,ref}$, expressed as Eq. (3).

$$K_{I\,ref} = \sigma\sqrt{\pi a}\left(1.12 - 0.23\left(\frac{a}{W}\right) + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4\right) \tag{3}$$

Theoretically, to validate the model, the Mode I SIF of the CIL model $K_{I \to CIL}$ must be close to the value of $K_{I\,ref}$ or $K_{I \to CIL} \approx K_{I\,ref}$, but it cannot be equal, i.e., $K_{I \to CIL} \neq K_{I\,ref}$. Therefore, based on Tada et al. (2001), the new $K_{I \to CIL}$ function is written as:

$$K_{I \to CIL} = \sigma^{\infty}\sqrt{\pi a_i}\left(C_1 - C_2\left(\frac{a}{W}\right) + C_3\left(\frac{a}{W}\right)^2 - C_4\left(\frac{a}{W}\right)^3 + C_5\left(\frac{a}{W}\right)^4 - C_n\left(\frac{a}{W}\right)^{n-1}\right) \tag{4}$$

where the constants $C_1, ...C_n$ are assumed to be constant over the crack surface. Shown in Figure 1(a) are two straight parallel edge cracks in a finite plate and the two cracks are under uniform far-field tension $P(\sigma^{\infty})$ normal to crack faces, and where the cracks are of equal lengths: a_1 and a_2. In this analysis, the ratios of (a/W) and (b/a) are used to evaluate the γ_I in the weak interaction region, which is assumed to be in the range of $1 \le b/a \le 3$. Therefore, the following cases are considered, where (b/a) = 1.5, 2.0, 2.5, 3.0 and (a/W) = 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50. Figure 1(a)–(c) also shows the two-dimensional configuration of the singular element. Based on Arakere et al. (2008), the meshing scheme is generated using ANSYS, as shown in Figure 1(d). The mesh encompassing the interacting cracks is divided into two zones: a global mesh, and the local mesh zone. Both zones are meshed with the 8-node isoparametric quadrilateral element that is used to build up the entire element for the two-dimensional plate, as illustrated in Figure 1(b). The SIF calculation is limited to the linear elastic problem with a homogeneous, isotropic material near the crack region. The studies are conducted in a pure Mode I loading condition with the specified material, Aluminium Alloy 7475 T7351 solid plate with constant thickness, homogenous isotropic continuum material, linear elastic behavior, small strain and displacements, and crack surfaces are smooth with the crack surfaces are almost contacting each other $(c_1 = c_2 = 0)$.

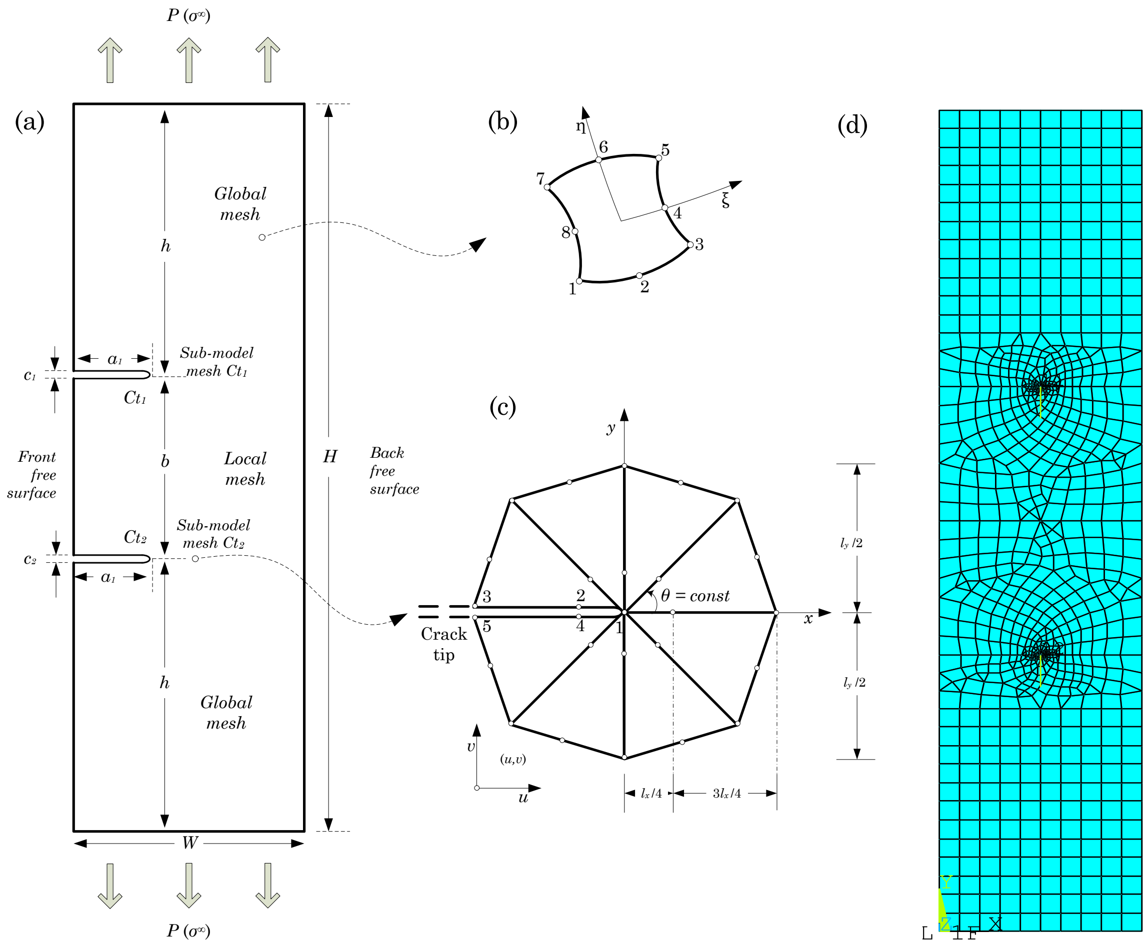


Figure 1. (a) Two parallel edge cracks in finite body, (b) 8-node quadrilateral element, (c) Barsoum singular element, and (d) complete meshing.

The SIF for CIL $K_{I\rightarrow CIL}$, is evaluated at the both crack tips of the edge strip cracks, as shown in Figure 1(a). The reduction of $K_{I\rightarrow CIL}$ comparative to $K_{I\,ref}$ is denoted as $\Delta K_{I\rightarrow CIL}$ and is expressed as $\Delta K_{I\rightarrow CIL} = K_{I\,ref} - K_{I\rightarrow CIL}$. The SIF for Mode I K_I, is determined using the displacement extrapolation method using written APDL macro code in ANSYS, and is expressed as Eq. (5).

$$K_{I\rightarrow CIL} = \left(E/3(1+\nu)(1+\kappa)\right)*\left(2\pi/l_y\right)^{1/2}*\left(4\left(\nu_2-\nu_4\right)-\left(\left(\nu_3-\nu_5\right)/2\right)\right) \quad (5)$$

where E=Young's Modulus, $\kappa = 3-4\nu$ for plane stress, $\kappa = 3-4\nu/1-\nu$ for plane strain, l_y is the length of element, v and u are displacements in a local Cartesian coordinate system, and υ is Poisson's ratio.
The elastic crack interaction factor for Mode I SIF is expressed as Eq. (6).

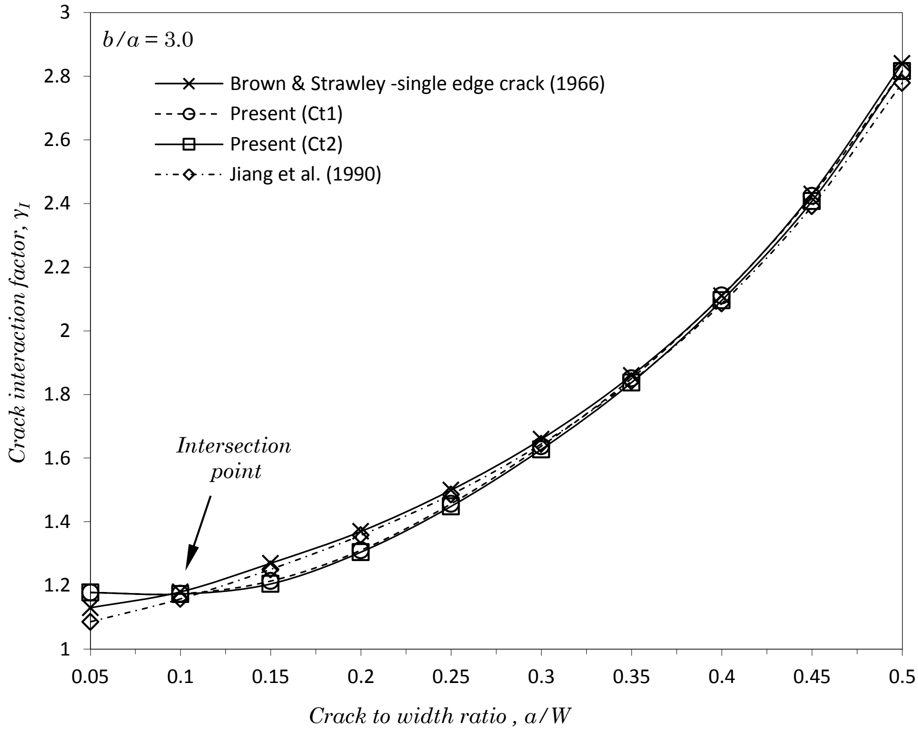
$$\gamma_I = K_{I\rightarrow CIL}/K_0 \quad (6)$$

where γ_I denotes the elastic interaction factor for Mode I and K_o is the SIF for the plain specimen.

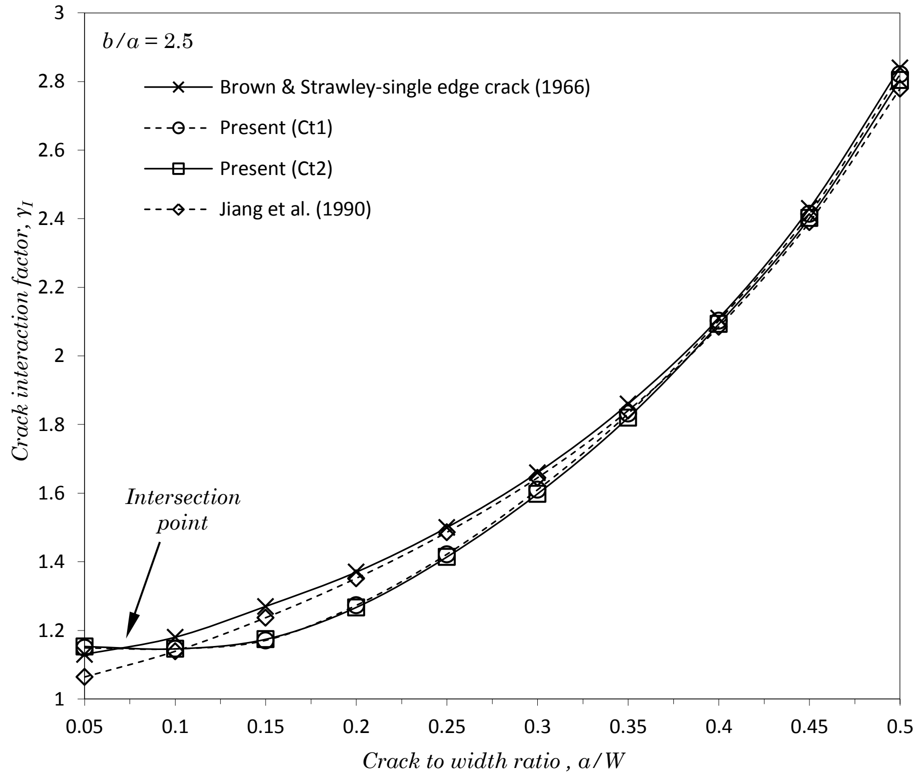
RESULTS AND DISCUSSION

Figure 2 shows the relationship between elastic crack interaction factor γ_I and the crack-to-width ratio a/W for the crack interval ratios $(b/a=3)$ and $(b/a=2.5)$. The elastic crack interaction factor γ_I is based on the SIF at the crack tips $K_{I\rightarrow CIL}$ and is normalized by K_0, which is the SIF for the plain specimen. In the weak interaction range, $(1<b/a\geq 3)$ for $(b/a=3)$, strong interaction γ_I appears at $a/W=0.5$ and it rapidly decreases when a/W approaches $a/W=0.05$. Therefore, the intensity of γ_I depends on the variation of a/W; it becomes larger as a/W increases. Figure 2 also shows a very small discrepancy (0.035%) between the intensity of γ_I for crack tips Ct_1 and Ct_2. This indicates that under the assumption of a homogenous material with equal length cracks, the intensity of γ_I can be assumed equal.

According to FFS codes by ASME (2004, 2007), BSI (1997, 2005), and JSME (2000, 2008), parallel cracks can be assumed a single coplanar crack under certain conditions of crack interval. In reduction of the a/W ratio from 0.5 to 0.05, the present numerical model supports the FFS code by the indication of the intersection point at 0.1 and 0.07 (see Figure 2) and 0.05 (see Figure 3) as the crack interval decreases from $b/a=3$ to $b/a=1.5$. The prediction by Jiang et al. (1990) of $F_n(K_I)$ shows no intersection point for a single crack of $f_I(K_{I\,ref})$ (Brown and Strawley, 1966), and shows no possible sign for single coplanar crack agreement, as outlined by the FFS codes. Again, the present $\gamma_{I\rightarrow CIL}(C_1)$ and $\gamma_{I\rightarrow CIL}(C_2)$ show a good trend line prediction of elastic crack interaction intensity and comply with the recommended FFS codes.

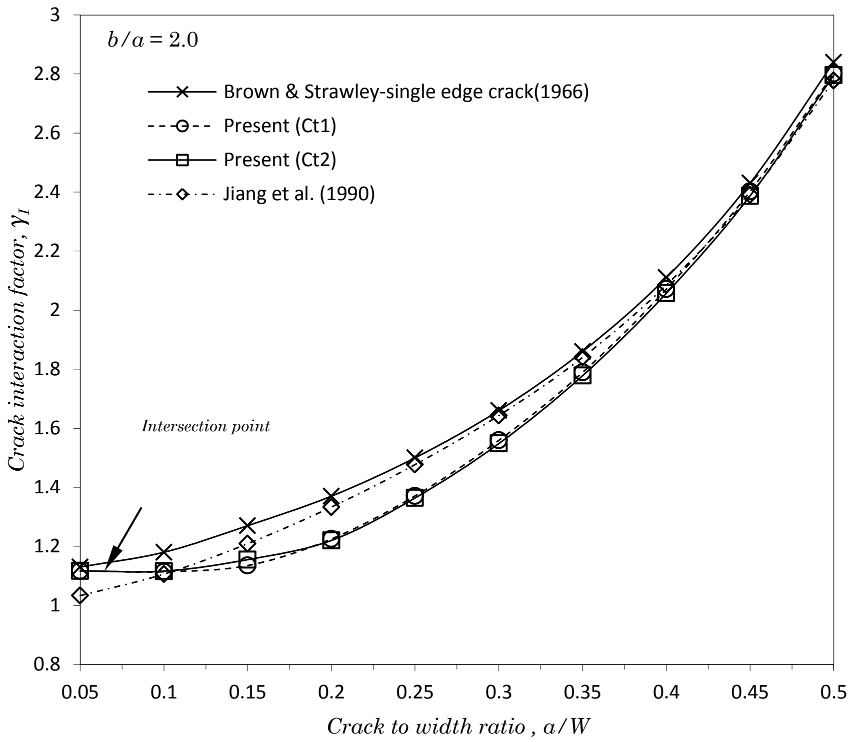


(a)

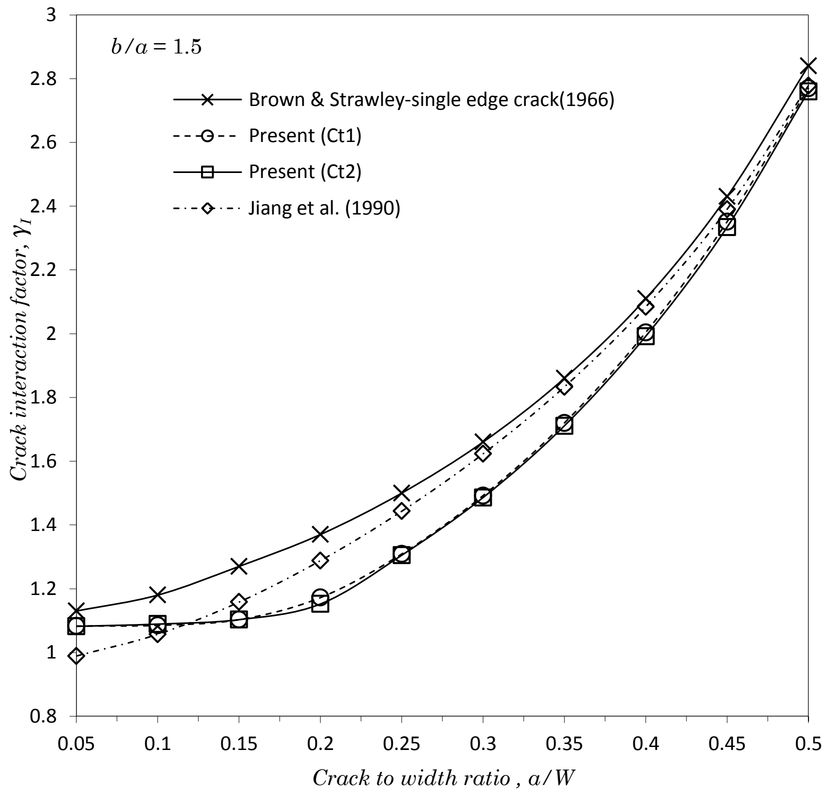


(b)

Figure 2. Elastic crack interaction factor: (a) $b/a = 3$, and (b) $b/a = 2.5$.



(a)



(b)

Figure 3. Elastic crack interaction factor: (a) $b/a = 2.0$, and (b) $b/a = 1.5$.

In order to validate the numerical model, the comparison is made with single crack SIF reference $K_{I\,ref}$ (Brown and Strawley, 1966) Eq. (3) expressed as $f_I(K_{I\,ref})$. Good agreement was obtained, which provides confidence on the FE modeling and CIL interaction analysis of shielding effects. When compared with the condition of CIL claimed by Jiang et al. (1990), denoted as $F_n(K_I)$, the present numerical model shows a significant improvement in the reduction of errors as a/W increases, as shown in Table 2.

Table 2. Crack interaction factor at $(b/a = 3.0)$.

a/W	Analytical	Analytical	Present	
	$f_I(K_{I\,ref})$	$F_n(K_I)$	$\gamma_{I\to CIL}(Ct_1)$	$\gamma_{I\to CIL}(Ct_2)$
0.05	1.133	1.085	1.178	1.178
0.1	1.184	1.158	1.174	1.173
0.15	1.266	1.251	1.214	1.205
0.2	1.373	1.358	1.311	1.305
0.25	1.505	1.486	1.457	1.449
0.3	1.665	1.645	1.638	1.628
0.35	1.864	1.842	1.851	1.838
0.4	2.113	2.086	2.111	2.096
0.45	2.432	2.392	2.424	2.408
0.5	2.843	2.799	2.817	2.816

CONCLUSION

An alternative solution for the elastic CIL for multiple parallel and equal edge strip cracks in a finite continuum body has been presented based on FE analysis. The close agreement with well-known analytical solutions for single edge cracks in a finite body validates the proposed solution for further applications for CIL analysis for closer crack distance, which involves higher crack interaction. Meanwhile, the compliance with recommended ASME codes, BSI codes, and JSME FFS codes provides additional evidence corroborating the present CIL prediction. Finally, it can be concluded that the CIL for equal and parallel edge cracks is depicted best at $b/a \geq 3$ because the crack interaction factor is approximately the normalized SIF value of a single edge crack with zero crack interaction.

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