

## CHEMICAL REACTION EFFECTS ON FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

**R. Muthucumaraswamy and V. Valliammal**

Department of Applied Mathematics,  
Sri Venkateswara College of Engineering  
Sriperumbudur 602 105, India.  
Email: [msamy@svce.ac.in](mailto:msamy@svce.ac.in)

### ABSTRACT

An analysis is performed to study the unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of a homogeneous chemical reaction of first-order. The plate temperature is raised linearly with time and the concentration level near the plate is raised uniformly. The dimensionless governing equations are solved using the Laplace transform. The velocity profiles are studied for different physical parameters such as the chemical reaction parameter, thermal Grashof number, mass Grashof number,  $a$ , and time. It is observed that the velocity increases with increasing values of  $a$  or  $t$ . But the trend is just the reverse in the chemical reaction parameter.

**Keywords:** chemical reaction, accelerated, vertical plate, exponential, heat transfer, mass diffusion.

### INTRODUCTION

Diffusion rates can be tremendously altered by chemical reactions. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface, and homogeneous if it takes place in solution. In most chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order if the rate of reaction is directly proportional to concentration. In many chemical engineering processes, there is a chemical reaction between a foreign mass and a fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production.

Chambre and Young (1958) have analysed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al. (1994) have studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The mass transfer effect on moving isothermal vertical plate in the presence of chemical reaction was studied by Das et al. (1999). The dimensionless governing equations were solved by the usual Laplace transform technique and the solutions are valid only at lower time level. Gupta et al. (1979) studied free convection in a flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using a perturbation method. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar (1984). The skin friction for an accelerated vertical plate has been studied

analytically by Hossain and Shayo (1986). Jha et al. (1991) analysed mass transfer effects on an exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy et al. (2009) studied an exact solution of flow past an accelerated infinite vertical plate with prescribed heat and mass flux. It is here proposed to study the unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of a first order chemical reaction. The dimensionless governing equations are solved using the Laplace transform. This study will be useful in chemical process industries such as wire drawing, fibre drawing, food processing and polymer production. The solutions are in terms of the exponential and the complementary error functions.

### ANALYSIS

The effects of a first order chemical reaction on the unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Here the  $x$ -axis is taken along the plate in the vertically upward direction and the  $y$ -axis is taken normal to the plate. At time  $t' \leq 0$ , the plate and fluid are at the same temperature  $T_\infty$ . At time  $t' > 0$ , the plate is exponentially accelerated with a velocity  $u = u_0 \exp(at')$  in its own plane and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is raised to  $C'_w$ . It is also assumed that there is a first order chemical reaction between the fluid and the species concentration. The reaction is assumed to take place entirely in the stream. Then under the usual Boussinesq approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_1 C' \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned} u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \leq 0 \\ t' > 0: \quad u = u_0 \exp(at'), \quad T = T_\infty + (T_w - T_\infty)At', \quad C' = C'_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{at } y \rightarrow \infty \end{aligned} \tag{4}$$

where  $A = \frac{u_0^2}{\nu}$ .

On introducing the non-dimensional quantities

$$U = \frac{u}{u_0}, \quad t = \frac{t'u_0^2}{\nu}, \quad Y = \frac{yu_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\begin{aligned} Gr &= \frac{g\beta v(T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - c'_\infty}, \quad Gc = \frac{vg\beta^*(C'_w - C'_\infty)}{u_0^3}, \\ Pr &= \frac{\mu C_p}{k}, \quad a = \frac{a'v}{u_0^2}, \quad K = \frac{vK_1}{u_0^2}, \quad Sc = \frac{v}{D} \end{aligned} \quad (5)$$

in (1) to (4), we obtain

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \quad (8)$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} U &= 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0 \\ t > 0: \quad U &= \exp(at), \quad \theta = t, \quad C = 1 \quad \text{at } Y = 0 \\ U &\rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{at } Y \rightarrow \infty \end{aligned} \quad (9)$$

### METHOD OF SOLUTION

Under the assumptions of this flow problem, it is observed that the energy equation (7) and the concentration equation (8) are uncoupled from the equation of momentum (6). It is clear that the temperature variable  $\theta(Y,t)$  and concentration equation  $C(Y,t)$  whereas  $U(Y,t)$  can be expressed in terms of  $\theta(Y,t)$  and  $C(Y,t)$ . The dimensionless governing equations (6) to (8) with the initial and boundary conditions (9) are tackled using Laplace transform technique. (Abramowitz and Stegun, 1972) and the results are as follows:

$$L(\theta) = \frac{e^{-Y\sqrt{sPr}}}{s^2} \quad (10)$$

$$L(C) = \frac{e^{-Y\sqrt{Sc(s+k)}}}{s^2} \quad (11)$$

$$\begin{aligned} L(U) &= \left(\frac{1}{s-a}\right)e^{-Y\sqrt{s}} + \left(\frac{Gr}{1-Pr}\right)\frac{1}{s^3} \left[e^{-Y\sqrt{sPr}} - e^{-Y\sqrt{s}}\right] \\ &+ \left(\frac{Gc}{1-Sc}\right)\frac{1}{s(S-b)} \left[e^{-Y\sqrt{Sc(s+k)}} - e^{-Y\sqrt{s}}\right] \end{aligned} \quad (12)$$

The inverse Laplace transform solutions of equations (10) to (12) are as follows:

$$\theta = t \left[ (1 + 2\eta^2 Pr) \operatorname{erfc}(\eta\sqrt{Pr}) - 2\eta\sqrt{\frac{Pr}{\pi}} \exp(-\eta^2 Pr) \right] \tag{13}$$

$$C = \frac{1}{2} \left[ \exp(2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) \right] \tag{14}$$

$$U = \frac{\exp(at)}{2} \left[ \exp(2\eta\sqrt{at}) \operatorname{erfc}(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at}) \operatorname{erfc}(\eta - \sqrt{at}) \right] + 2e \operatorname{erfc}(\eta) - \frac{dt^2}{6} \left[ (3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) - (3 + 12\eta^2 Pr + 4\eta^4 Pr^2) \operatorname{erfc}(\eta\sqrt{Pr}) + \frac{\eta\sqrt{Pr}}{\sqrt{\pi}} (10 + 4\eta^2 Pr) \exp(-\eta^2 Pr) \right] - e \exp(ct) \left[ \exp(2\eta\sqrt{ct}) \operatorname{erfc}(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct}) \operatorname{erfc}(\eta - \sqrt{ct}) \right] - e \left[ \exp(2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) \right] + e \exp(ct) \left[ \exp(2\eta\sqrt{Sc(K+c)t}) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{(K+c)t}) + \exp(-2\eta\sqrt{Sc(K+c)t}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{(K+c)t}) \right] \tag{15}$$

where  $c = \frac{KSc}{1-Sc}$ ,  $d = \frac{Gr}{1-Pr}$ ,  $e = \frac{Gc}{2c(1-Sc)}$ , and  $\eta = \frac{Y}{2\sqrt{t}}$ .

### RESULTS AND DISCUSSION

In order to get some physical insight into the problem, numerical computations were carried out for different physical parameters  $a$ ,  $Gr$ ,  $Gc$ ,  $Sc$  and  $t$  upon the nature of the flow and transport. The value of the Schmidt number  $Sc$  is taken to be 0.6 which corresponds to water–vapour. Also, the value of Prandtl number ( $Pr$ ) is chosen such that it represents air ( $Pr = 0.71$ ). The numerical values of the velocity are computed for different physical parameters like  $a$ , Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, and time. The effect of velocity for different times ( $t = 0.2, 0.4, 0.6, 0.8$ ),  $k = 2$ ,  $a = 0.5$ ,  $Gr = Gc = 5$  are shown in Figure 1. In this case, the velocity increases gradually with respect to time  $t$ . Figure 2 illustrates the effect of velocity for different values of the chemical reaction parameter ( $k = 0.2, 2, 5$ ),  $a = 0.5$ ,  $Gr = 5$ ,  $Gc = 10$  and  $t = 0.2$ . The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the relative variation of the velocity with the magnitude of the chemical reaction parameter. The velocity profiles for different ( $a = 0, 0.2, 0.5, 0.8$ ),  $k = 2$ ,  $Gr = Gc = 5$  at  $t = 0.2$  are studied and presented in Figure 3. It is observed that the velocity increases with increasing values of  $a$ .

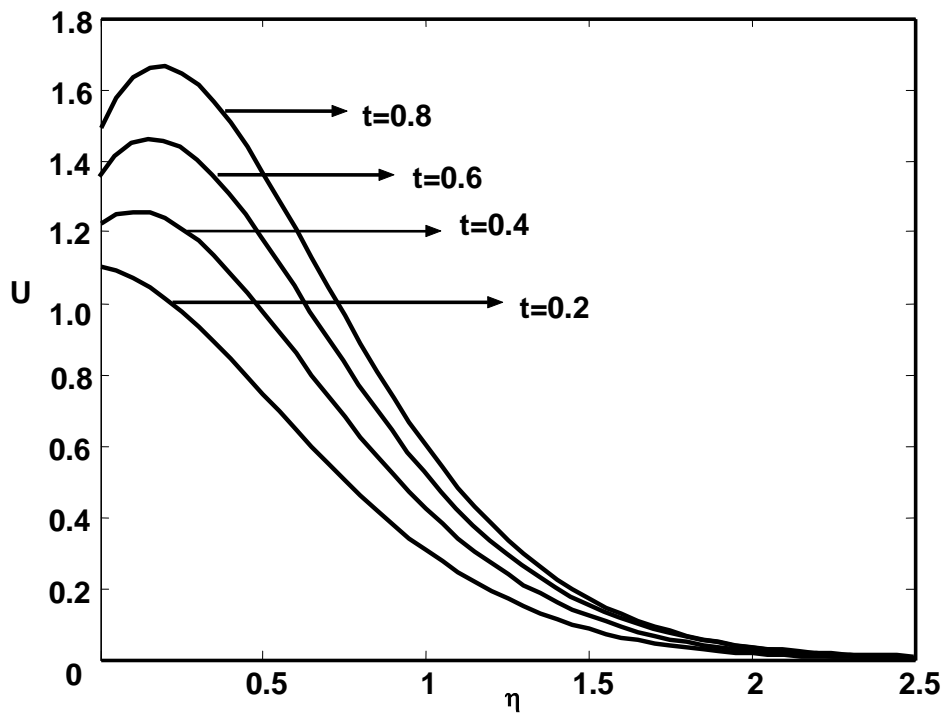


Figure 1. Velocity profiles for different values of  $t$

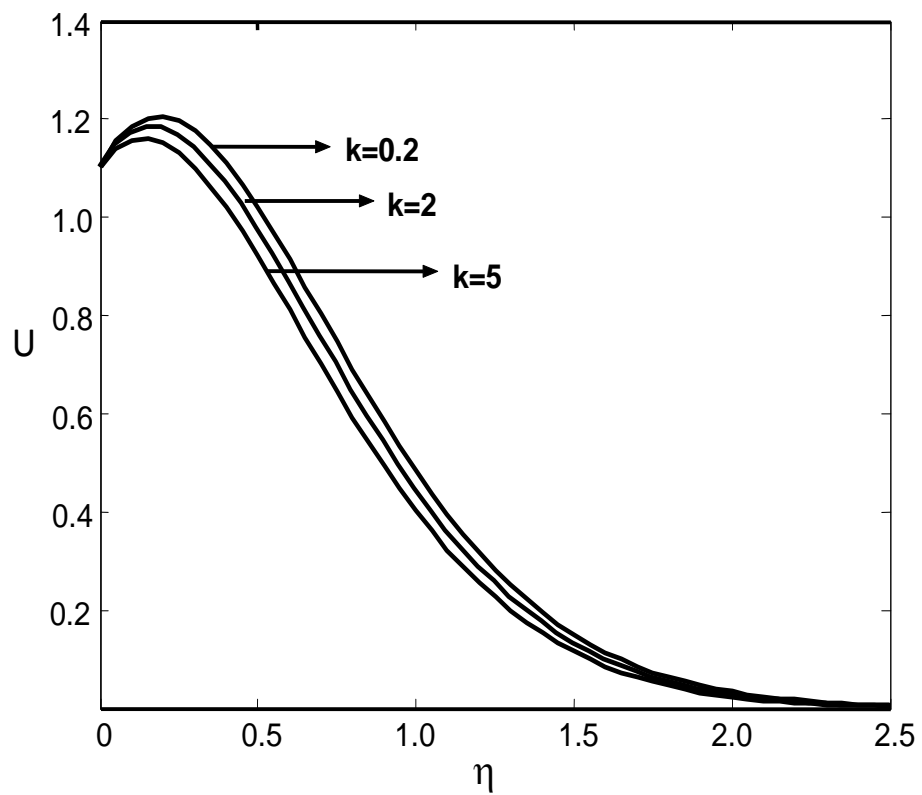


Figure 2. Velocity profiles for different values of  $k$

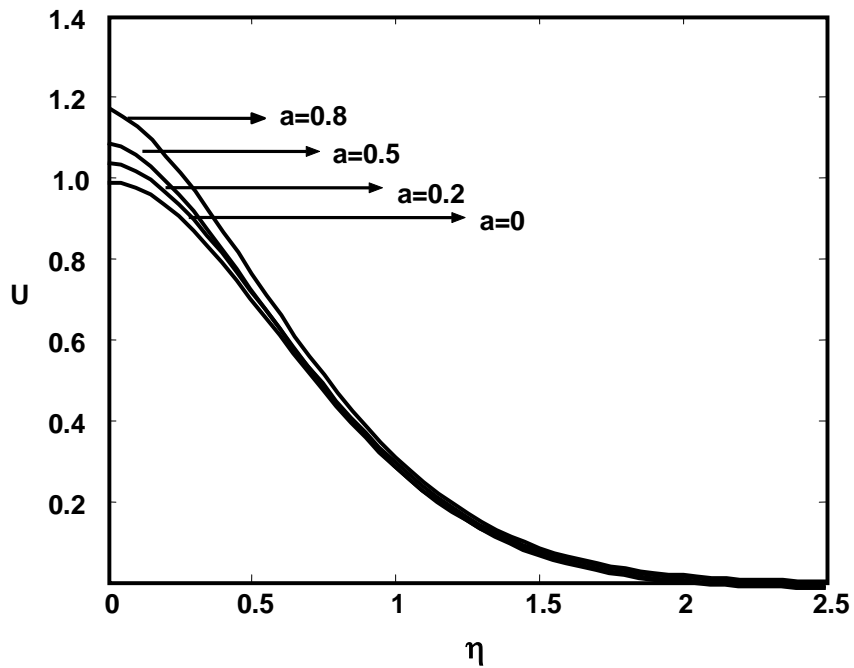


Figure 3. Velocity profiles for different values of  $a$

Figure 4 represents the effect of concentration profiles at time  $t = 0.2$  for different Schmidt number ( $S_c = 0.2, 0.3, 0.6, 0.8$ ). The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to zero far away in the free stream. It is observed that the wall concentration increases with decreasing Schmidt number.

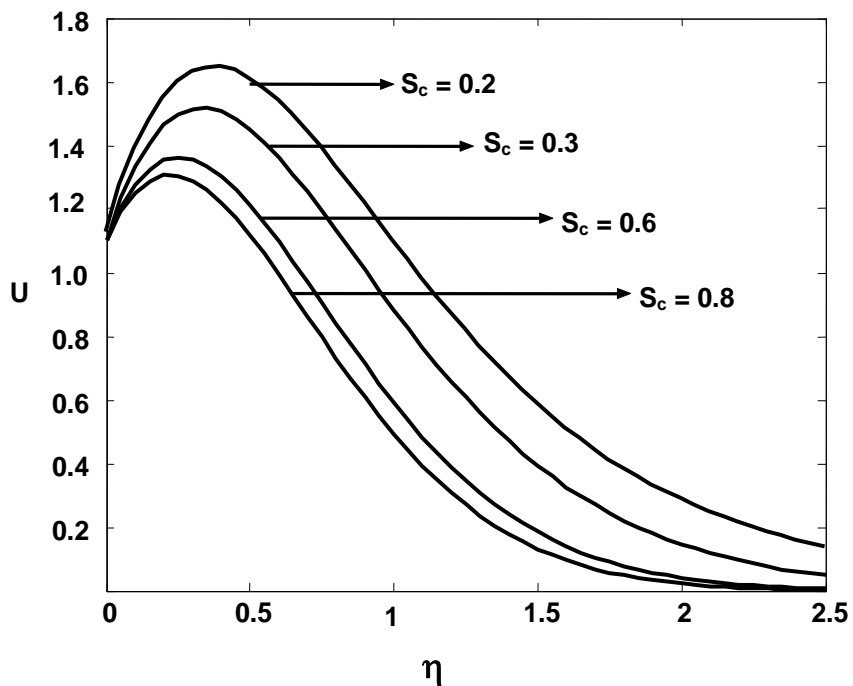


Figure 4. Velocity profiles for different values of  $S_c$

## CONCLUSION

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations were solved by the usual Laplace transform technique. The effect of different parameters such as the thermal Grashof number, mass Grashof number,  $a$  and  $t$  were studied graphically. It was observed that the velocity increases with increasing values of  $Gr$ ,  $Gc$ ,  $a$  and  $t$ . But that trend is reversed with respect to the Schmidt number.

## REFERENCES

- Jha, B.K., Prasad R. and Rai, S. (1991) Mass Transfer Effects on the Flow Past an Exponentially Accelerated Vertical Plate With Constant Heat Flux. *Astrophysics and Space Science*, 181:125–134.
- Chambre, P.L. and Young, J.D. (1958) On the Diffusion of a Chemically Reactive Species in a Laminar Boundary Layer Flow. *The Physics of Fluids*, 1: 48–54.
- Das, U.N. Deka, R.K. and Soundalgekar, V.M. (1994) Effects of Mass Transfer on Flow Past an Impulsively Started Infinite Vertical Plate With Constant Heat Flux and Chemical Reaction. *Forschung im Ingenieurwesen*, 60: 284–287.
- Das, U.N. Deka, R.K. and Soundalgekar V.M. (1999) Effects of Mass Transfer on Flow Past an Impulsively Started Infinite Vertical Plate With Chemical Reaction. *The Bulletin of GUMA*, 5: 13–20.
- Gupta, A.S., Pop, I. and Soundalgekar, V.M. (1979) Free Convection Effects on the Flow Past an Accelerated Vertical Plate in an Incompressible Dissipative Fluid. *Rev. Roum. Sci. Techn.-Mec. Apl.*, 24: 561–568.
- Hossain, M.A. and Shayo, L.K. (1986) The Skin Friction in the Unsteady Free Convection Flow Past an Accelerated Plate. *Astrophysics and Space Science*, 125: 315–324.
- Muthucumaraswamy R., Sundar Raj M. and Subramanian V.S.A. (2009) Exact Solution of Flow Past an Accelerated Infinite Vertical Plate With Heat and Mass Flux, *International Journal of Applied Mechanics and Engineering*, 14: 585–592
- Singh, A.K. and Kumar, N. (1984) Free Convection Flow Past an Exponentially Accelerated Vertical Plate, *Astrophysics and Space Science*, 98: 245–258.

## NOMENCLATURE

A, a	constants
$C'$	species concentration in the fluid, $\text{mol} \oplus \text{m}^{-3}$
C	dimensionless concentration
$C_p$	specific heat at constant pressure, $\text{J} \oplus \text{kg}^{-1} \oplus \text{K}^{-1}$
D	mass diffusion coefficient, $\text{m}^2 \oplus \text{s}^{-1}$
$Gc$	mass Grashof number
$Gr$	thermal Grashof number
g	acceleration due to gravity, $\text{m} \oplus \text{s}^{-2}$
k	thermal conductivity, $\text{J} \oplus \text{m}^{-1} \oplus \text{K}^{-1}$
$K_t$	chemical reaction parameter, J
K	dimensionless chemical reaction parameter

Pr	Prandtl number
Sc	Schmidt number
T	temperature of the fluid near the plate K
t'	time, s
t	dimensionless time
u	velocity of the fluid in the x-direction, $\text{m}\oplus\text{s}^{-1}$
$u_0$	velocity of the plate, $\text{m}\oplus\text{s}^{-1}$
U	dimensionless velocity
x	spatial coordinate along the plate
y'	coordinate axis normal to the plate, m
y	dimensionless coordinate axis normal to the plate

### Greek symbols

$\beta$	volumetric coefficient of thermal expansion, $\text{K}^{-1}$
$\beta^*$	volumetric coefficient of expansion with concentration, $\text{K}^{-1}$
$\mu$	coefficient of viscosity, $\text{Pa}\oplus\text{s}$
$\nu$	kinematic viscosity, $\text{m}^2\oplus\text{s}^{-1}$
$\rho$	density of the fluid, $\text{kg}\oplus\text{m}^{-3}$
$\tau$	dimensionless skin-friction
$\theta$	dimensionless temperature
$\eta$	similarity parameter
erfc	complementary error function

### Subscripts

w	conditions at the wall
$\infty$	conditions in the free stream