

FINITE ELEMENT ANALYSIS OF HEAT TRANSFER IN MULTI-LAYER COOKING POTS WITH EMPHASIS ON LAYER NUMBER

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ABSTRACT

The goal of this research was to numerically assess the dependence of cookware performance on the number of layers and materials as defined by the uniform surface temperature and average temperature on the cooking surface. The objectives of this research were: to quantify the temperature distribution in items of cookware based on variation of the number of layers and thermal properties by using a finite element model of heat transfer in cookware. 625 types of composite cookware were compared in terms of thermal performance. The results show that a higher thermal conductivity material yields a more uniform surface temperature profile than a lower thermal conductivity material. In addition, cookware with a two-ply base provides a more uniform temperature profile and heat transfer compared with one, three and four layers. Low conduction materials used in a thin second layer with conductive metals as a first layer reduce the temperature differences up to 180 K. Moreover, this combination with high conduction metals used in a thick first layer provides a high mean temperature on the non-heated surface.

Keywords: Cookware; conduction; finite element method; multi-layer plate.

INTRODUCTION

An ideal cookware should be designed to get the maximum heat from the burner to increase its efficiency and economize on energy. Besides, it should meet the majority of consumer demands including being lightweight and durable, keeping the contents hot for a relatively long time, avoiding hot spots on the cooking surface and having a non-stick surface. It also has to guarantee the health of the consumers, and so must not react with the contents. We can satisfy a wide variety of demands including superior mechanical, chemical and thermal properties by using multiple materials together [1, 2]. One way of reducing house energy consumption is to design heating facilities, which are more economical in their use of energy [3]. Generally, a multi-layer structure and the material properties of the layers have a high impact on improving the thermal behavior of cookware, and can optimize the energy consumption. The energy is obtained mainly from burning gas and electrical resistivity. The heat is not uniformly spread over the pan in either method. Using a multi-layer plate provides regular temperature distribution on the top when the bottom is heated unevenly [4-6]. There are a few academic papers that have used experimental or numerical methods to study cookware performance. A computer code, finite difference, was developed to study a

single pan stove [7]. Ashman, Junus [8] used an experimental method to study the behavior of efficiency and pollution dissipation from the burning head. Sabilov, Farkas [9] used a finite element method to simulate conduction heat transfer through the dish wall. They also studied the effect of conductivity on cooking quality. An analytical model was used by Jugjai and Rungsimuntuchart [10] to simulate convection heat transfer from the burning head to the dish. They found the highest efficiency by using a swirling central flame. Lucky and Hossain [11] conducted an experimental research on Bangladeshi cookstoves. They found that a pan is more efficient than a pot. Karzar Jeddi, Kazemzadeh Hannani [12] used the finite element method to model heat transfer through burners to the pan. Ayata [13] used the finite element, ANSYS program, to model the temperature distributions in the copper and aluminum layered base of a chromium nickel saucepan. Sedighi and Dardashti [14] studied the dependence of heat transfer on materials. In the current work, a numerical simulation of the system has been carried out using the finite element method to study the dependence of heat transfer on the thicknesses and number of layers, the materials and geometric properties. The authors believe that the thermal analysis of multi-metal cookware with one to four layers and different materials is applied here for the first time. The intention of this investigation is to try to bridge the information gap.

MATERIALS AND METHODS

The annular part of the circular surface of the bottom side of a plate illustrated as r in Figure 1, was constrained to a fixed temperature of 773 (K) to model an irregular heat source. There is a geometrical symmetry so the system can be modeled by a rectangular plane with the length of the pan radius and a thin and long rectangle as the wall of the pan. Because of the symmetry, the temperature gradients at the center of the plate along the y -axis have zero value. Hence there is no heat flux at the center of the plate along the y -axis. The side of the pan has convection heat transfer with air at ambient temperature. The thickness of the plate is 10 mm.

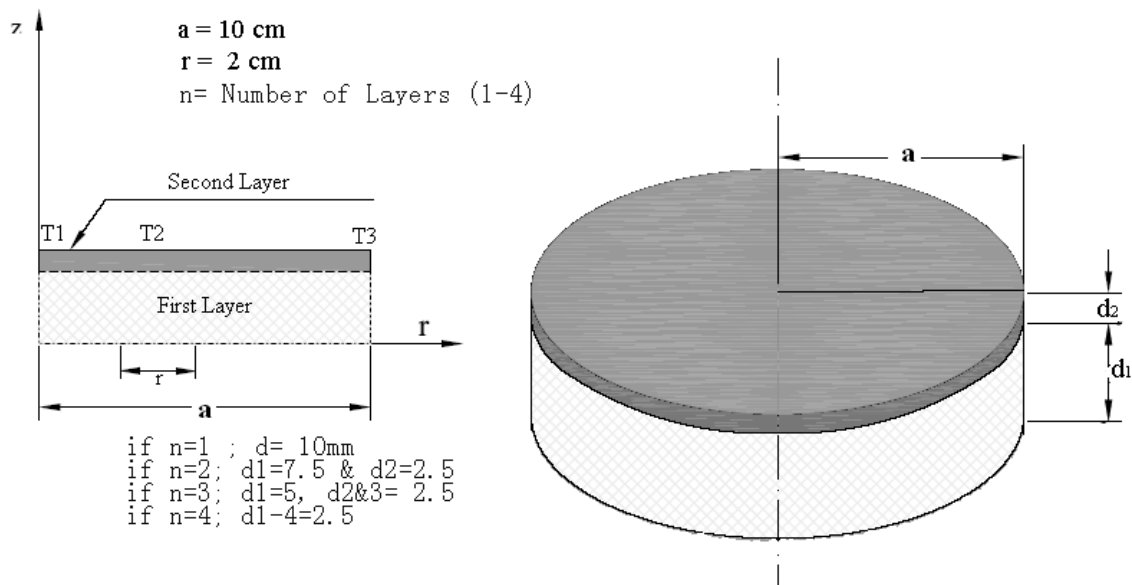


Figure 1. 2D and 3D schematic of section of a circular bi-layer plate.

The analysis has been extended to four, three and two layers as well as a single layer. In four layers, all layers have the same thickness of 2.5 mm. In three layers, one layer is 5 mm and the other two are 2.5 mm. For two layers, the thicknesses are 7.5 and 2 mm. The ambient temperature and the coefficient of heat transfer have been assumed as 293 K and 17 W/m².K, respectively. In addition, it is also assumed that the pan is filled with water at boiling temperature, and the coefficient of heat transfer between the pan and the water is 50 W/m².K. Copper (Cu), aluminum (Al), chromium nickel (CrNi), titanium (Ti) and stainless steel (SSt) have been applied in each layer. A loop was defined to change the five materials through the four layers. Overall, 625 models were analyzed. The properties of the applied metals are according to Bergman and Incropera [15].

Numerical Method

In the finite element method, a given computational domain is subdivided as a collection of a number of finite elements, subdomains of variable size and shape, which are interconnected in a discrete number of nodes. The solution of the partial differential equation is approximated in each element by a low-order polynomial in such a way that it is defined uniquely in terms of the solution at the nodes. The global solution can then be written as series of low-order piecewise polynomials with the coefficients of the series equal to the approximate solution at the nodes [16]. Solution of the two-dimensional heat analysis finite element method is governed by [17]:

$$\{g\} = \begin{Bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial x} & \frac{\partial N_m}{\partial x} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_j}{\partial y} & \frac{\partial N_m}{\partial y} \end{bmatrix} \begin{Bmatrix} t_i \\ t_j \\ t_m \end{Bmatrix} \tag{1}$$

$$[B] = \frac{\partial}{\partial x} [N] = \frac{1}{|x|} \begin{bmatrix} \beta_i \beta_j \beta_m \\ \gamma_i \gamma_j \gamma_m \end{bmatrix} \tag{2}$$

where analogous to a strain matrix $\{g\}=[B]\{t\}$ and $[B]$ is a derivative of $[N]$. The heat flux and temperature gradient are written as:

$$\begin{Bmatrix} q_x \\ q_y \end{Bmatrix} = - \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{yy} \end{bmatrix} \{g\} = -[D]\{g\} \tag{3}$$

The first term of Equation (4) is the conduction portion and the second term is the convection portion of the total stiffness matrix.

$$[k] = \iiint_v [B]^T [D][B] dV + \iint_s h[N]^T [N] dS \tag{4}$$

where the heat source is constant, we have:

$$\{f_o\} = Q \iiint_v [V]^T dV = \frac{QV}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \tag{5}$$

$$\{f\} = [k]\{t\} \tag{6}$$

The stiffness matrix is a general term for a matrix of known coefficients being multiplied by unknown degrees of freedom, i.e., temperature, etc. Thus, the element conduction matrix is often referred to as the stiffness matrix.

$$\{F\} = [K]\{t\} \tag{7}$$

Heat flux boundary conditions are already accounted for in the derivation. We just substitute into the above equation and solve for the nodal temperature and element temperature gradient [17].

RESULTS AND DISCUSSION

In this part, copper, aluminum, chromium nickel, titanium and stainless steel were used in each layer to determine the behavior of the two parameters of mean temperature (Tmean) and temperature differences (DT=Tmax-Tmin) on the cooking surface of the plate. Figure 2 illustrates the mean temperature on the cooking surface of the cookware. The maximum point occurs when all four layers are copper. This means that the copper single-layer plate provides the highest mean temperature. As shown in Figure 2, the mean temperature on the cooking surface increases with the use of conductive metals. Hence aluminum, compared with titanium, stainless steel and chromium nickel, provides the highest mean temperature.

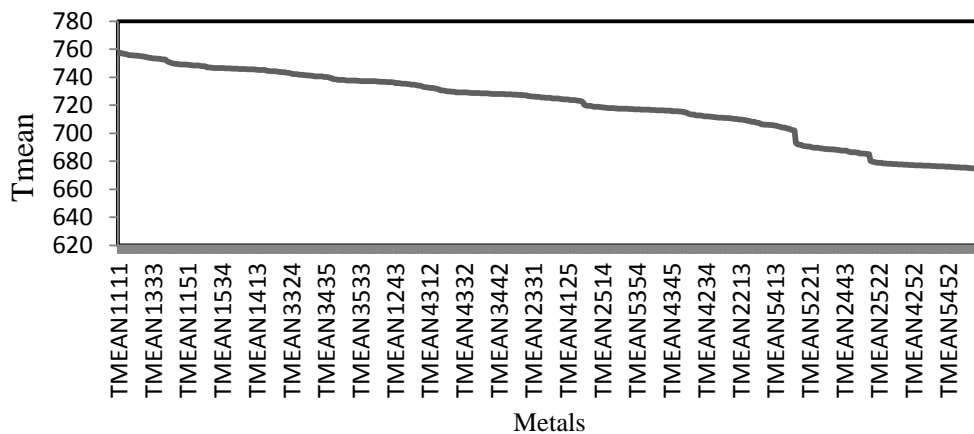


Figure 2. Mean temperature on cooking surfaces of multi-metal plates.

*The order of numbers indicates the materials of the first to fourth layers. The numbers indicate the metals as follows:

- 1=Cu
- 2=SSt
- 3=Al
- 4=CrNi
- 5=Ti

The plates whose first layers are conductive metals such as copper and aluminum show clearly different behavior from the plates whose first layers are low-conduction metals. The mean temperatures in plates whose first layer is a low-conduction metal are approximately the same. The maximum temperatures of the stainless steel, chromium nickel and titanium are 737.3 K, 735.7 K and 735.15 K respectively. The difference of the maximum and minimum temperatures on the top surface of the multi-metal plates is illustrated in Figure 3. The minimum temperature difference occurs with $P_{Cu/CrNi}$ (P_{1114} or $P_{Cu/Cu/Cu/CrNi}$), and is as low as 35.2 K. This means that the bi-layer plate with Cu and CrNi layers with thicknesses of 7.5 mm and 2.5 mm respectively provides the most uniform temperature distribution on the cooking surface. Plates whose first layers are conductive metals such as copper and aluminum have lower temperature differences than plates whose first layers are low-conduction metals. The maximum DT was provided by $P_{CrNi/Ti}$ at 216 K.

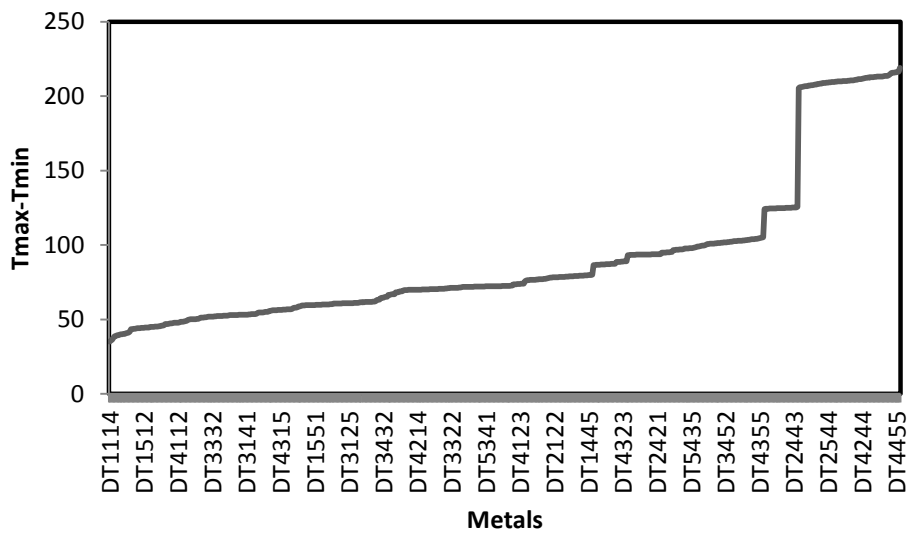


Figure 3. Temperature differences on cooking surfaces of multi-metal plates.

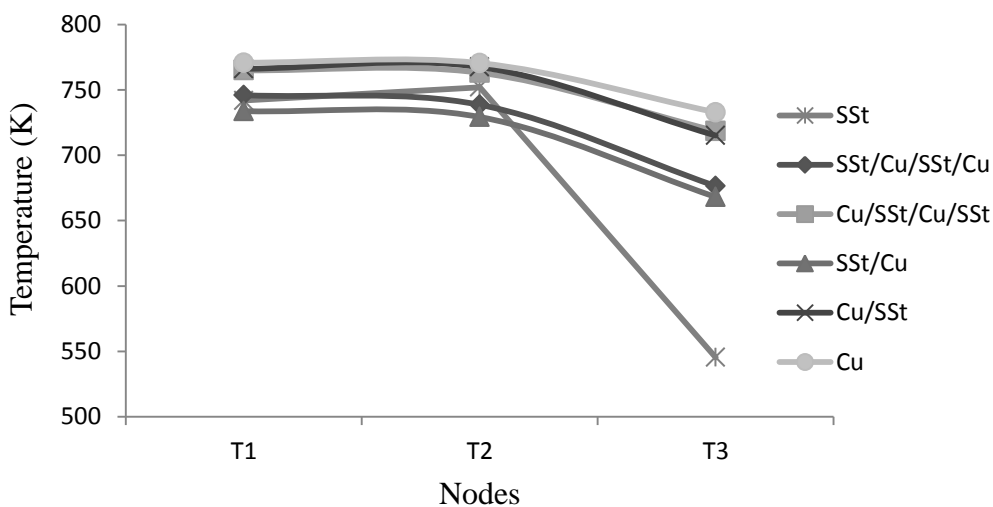


Figure 4. Temperature distribution on cooking surfaces of single and multi-metal plates consisting of copper and stainless steel.

Cookware manufacturers usually clad non-reactive and resistant metal, namely stainless steel, exteriors around a core made out of a more conductive material, such as copper. Figure 4 compares the temperature distribution through the one to four layers consisting of copper and stainless steel. The bi-layer plate consisting of copper (7.5 mm) and stainless steel (2.5 mm) provides the most uniform cooking surface. Also the figure demonstrates that the maximum temperature difference occurs with single-layer stainless steel due to its poor heat conduction. The data in Table 1 has been selected from among the 625 points (models) of Figures 2 and 3 which have higher values of the two parameters than others. According to this table, the single-layer plate of copper provides the highest mean temperature on the cooking surface. However, the bi-layer plates including $P_{Cu/CrNi}$, $P_{Cu/SSt}$, $P_{Cu/Al}$, $P_{Cu/Ti}$ and three-layer plates including $P_{Cu/Al/Cu}$, $P_{Cu/Al/CrNi}$ and $P_{Al/Cu/SSt}$ provide high mean temperatures also. The lower the temperature difference (DT), the higher the uniformity will be. Based on Table 1, the $P_{Cu/CrNi}$ bi-layer plate provides the most uniformity on the cooking surface. Thereafter, the $P_{Cu/SSt}$, $P_{Cu/Ti}$, P_{Cu} single-layer plates and the four-layer plate of $P_{Cu/Al/Cu/SSt}$ have the next highest uniformity respectively.

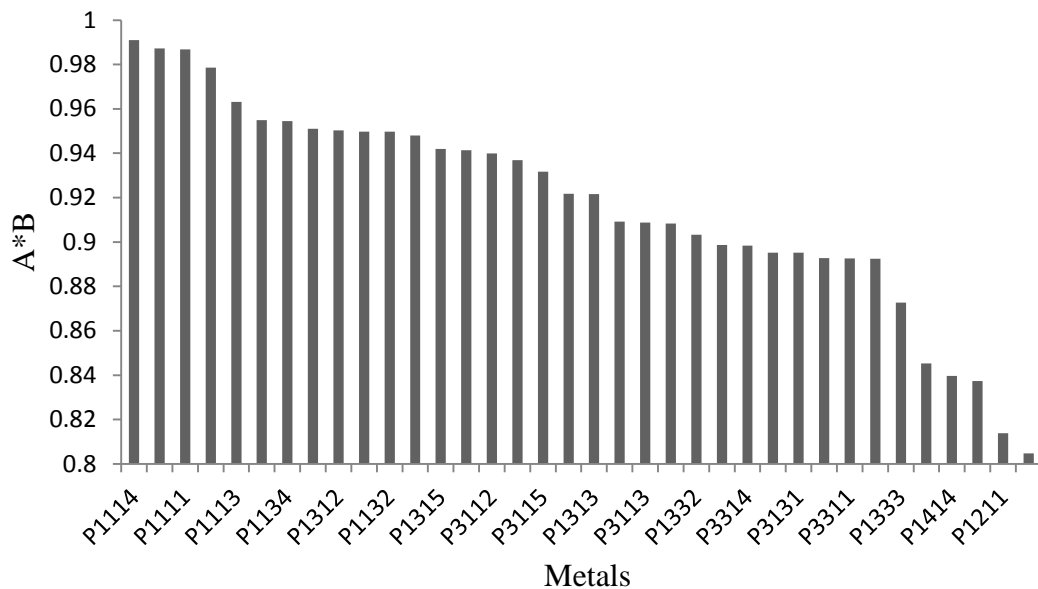


Figure 5. Normalized mean temperature and temperature differences of Table 1. A=normalized mean temperature; B=normalized temperature difference.

Figure 5 demonstrates the normalized data of Table 1 to find the plate which provides the highest T_{mean} and lowest DT. According to the figure, the bi-layer cookware of $P_{Cu/CrNi}$ has the best performance against the two parameters of mean temperature and uniformity. After this, the $P_{Cu/SSt}$ and $P_{Cu/Ti}$ bi-layers, P_{Cu} single-layer, the $P_{Cu/Ti}$ and $P_{Cu/Al}$ two-ply and the four-layer plate of $P_{Cu/Al/Cu/SSt}$ show satisfactory behavior with respect to the two parameters as cookware. These results match the research results of Ayata, Çavuşoğlu [18], who applied numerical modeling of a CrNi saucepan with layered bases of different alloys of aluminum and copper. They found that copper performs better than aluminum as the bottom layer.

Table 1. Mean temperature and temperature differences on cooking surface of selected plates.

Metals*	Tmean	DT	Metals*	Tmean	DT
1111	758.0082	37.61687	1331	754.782	45.38596
1114	757.2483	35.19641	3113	754.6477	45.03116
1112	757.2213	35.83858	3115	754.6413	40.63268
1113	757.2164	40.28539	1334	754.0766	43.71586
1115	756.7444	36.40339	1332	753.9688	44.59857
1131	756.6134	41.24785	3131	753.9323	46.10216
1311	756.3964	41.01728	3311	753.6231	45.91908
1134	755.8908	39.04713	1333	753.5736	49.68836
1132	755.8272	39.80419	1335	753.499	45.14755
1314	755.7871	38.72077	3134	753.4489	44.37377
1312	755.7236	39.46157	3132	753.329	45.26532
1133	755.6343	44.66116	3314	753.2758	44.05708
3111	755.586	41.69657	3312	753.1544	44.93259
1313	755.4528	44.31465	1214	749.0477	45.14755
1135	755.3544	40.3582	1414	748.596	45.26532
1315	755.252	40.01501	1514	748.4439	45.38596
3114	755.1855	38.72077	1211	746.5662	45.91908
3112	755.1099	40.08198	1411	745.8267	46.10216

CONCLUSIONS

Two-dimensional finite element analysis of heat transfer in multi-layer plates when heating non-uniformly was investigated at steady-state conditions. The analysis was extended to include varying numbers of layers, thicknesses of layers and materials. We applied Cu, Al, SSt, CrNi and Ti in one to four layers in order to find the two parameters of mean temperature and uniformity on the cooking surfaces of the plates. In this part of the study, 625 models were analyzed. The results clearly showed that, when the first layer which is exposed to heat is a conductive metal like copper or aluminum, the cookware demonstrates a higher mean temperature and greater uniformity on the cooking surface. Through investigation of the 625 models with one to four layers, we found that bi-layer plates of $P_{Cu/CrNi}$ and $P_{Cu/SSt}$ provide the best performance as cookware. All-clad copper and aluminum plates have lower temperature gradients than single-layer aluminum and all-clad aluminum core plates. Also, single-layer stainless steel is unsuitable to use as cookware due to its poor heat conduction.

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NOMENCLATURE

DT	$T_{\max}-T_{\min}$ (K)	Cu	copper
$\{f\}$	force matrix	Al	aluminum
$\{g\}$	gradient matrix	$CrNi$	chromium nickel
$[K]$	element stiffness matrix	Ti	titanium
N	shape function	SSt	stainless steel
Q	constant heat source		
q	heat flux		
T_{mean}	average temperature (k)		

Subscripts

c	conduction
h	convection
P	plate