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PREDICTION OF DYNAMIC CHARACTERISTICS OF VIBRATING STRUCTURE WITH UNCERTAIN PARAMETERS

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ABSTRACT

This paper is based on a study of vibrational responses of different vibrating systems with uncertain parameters. The concept of uncertainty plays an important role in the design of the practical mechanical system. So it becomes important to study its effects on the mechanical system. A large and varied amount of research has been dedicated to developing techniques which predict the dynamic responses of structures in all frequency domains and structures with uncertainty. The structural element selected in this work is the plate. The plate is considered in terms of mass, stiffness and a combination of mass and stiffness uncertainty. The dynamic characteristic of the plate with all uncertain parameters is found by using the Finite Element Method. In this paper modal and harmonic analysis of the plate is done. In modal analysis the natural frequencies and mode shapes of the plate are found. The response of a bare plate is compared with plates for which different uncertain parameters are considered. Similarly, in harmonic analysis the frequency response function of the bare plate and the plates with all uncertain parameters are compared. During comparison, it was found that due to mass uncertainty minor changes in the natural frequency and in mode shape were obtained. However, due to stiffness the natural frequency, mode shape and FRF were drastically changed. Similarly, due to a combination of mass and stiffness uncertainty drastic changes were observed in the plate response. Because of these uncertainties the complete vibrational characteristics were changed. So it becomes important to consider these uncertainties to avoid misinterpretation while designing plates.

Keywords: Uncertain Parameters; vibration analysis; modal analysis; harmonic analysis; FEM.

INTRODUCTION

Predicting the dynamic response of a vibrating system generally involves determining the equations of motion of the structure and solving them in order to find the natural frequencies and mode shapes of the system to give boundary conditions. The natural frequencies and mode shapes can then be used to predict the response due to an applied excitation [1-3]. For more complex systems, the equations of motion can be approximated using various deterministic modelling techniques such as finite element analysis (FEA) and dynamic stiffness techniques [4-10]. These methods are extensively used to predict the linear dynamic response of structures in the low frequency region. Energy methods such as Statistical Energy Analysis (SEA) are appropriate dynamic predictive techniques at high frequencies [11, 12]. In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of design parameters. The study of mathematical models involves physical and geometric parameters such as mass density ρ, elastic modulus E, Poisson's ratio v, lengths, and cross-section shape characteristics [13]. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution, geometric properties or physical errors, as well as variation arising from assembly and manufacturing processes [14, 15]. For many dynamic structures it is not possible to know their exact material properties or geometry. This uncertainty can be due to a number of factors such as variation in material properties, structural dimensions or changes in excitation over time. Changes in excitations over time can be caused by wear or fatigue. For example, the wearing of gear teeth or the development of cracks in the teeth can affect the frequency and the amplitude of the harmonic excitations caused by these rotating components. Because of this, it is very important to study and investigate the effect of uncertainty on the overall dynamic characteristics of the structure [16-19].

Vibroacoustic tests were carried out by Kompella and Bernhard [20] on 98 similar vehicles. They measured both airborne and structure-borne transfer paths across the fleet of vehicles and found that both responses varied by up to 20 dB in the high-frequency range. For many vibratory systems, uncertainty can have a significant effect on the vibrational response even at low frequencies. Wood and Joachim [21], [22] showed that for a four cylinder car the scatter in structure-borne interior noise can vary by as much as 15 dB. Cornish [23] found great variability in the vibroacoustic responses of five identical vehicles across the entire frequency range from 100 Hz to 400 Hz.In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an important role in the investigation of various engineering problems [13].

The uncertainty concerned in this paper is the variation in material properties such as mass and stiffness. The response of any system with uncertainty will differ from the response of a system model. The level of variation of the response will depend on the degree of uncertainty. At low frequencies, the effect is often negligible, but as the frequency increases, the effect becomes more significant. In this paper modal and harmonic analysis is done to find and compare the response of a bare plate and plates for which all uncertain parameters are considered.

MATHEMATICAL MODELLING

All the mechanical structures such as machines, vehicles, aircraft, home appliances and civil engineering structures are made up of plate or combination of plates. Plates possess uncertainty during their manufacturing. It becomes necessary to study the vibration of plate with and without uncertainty. Using the Lagrange–Rayleigh–Ritz technique [24], the equations of motion of a dynamic plate in modal space can be derived. Initially a bare plate dynamic is derived and then the addition of different uncertainty in the plate is derived.

Bare Rectangular Plate

The simply supported rectangular bare plate (with no structural uncertainty) is considered as shown in Figure 1. The Lagrange–Rayleigh–Ritz technique is applied to a range of dynamic systems in order to examine the natural frequency statistics. Using the Lagrange–Rayleigh–Ritz technique, the equations of motion of a dynamic system in modal space can be derived. This technique is used to obtain the natural frequencies of the bare plate.

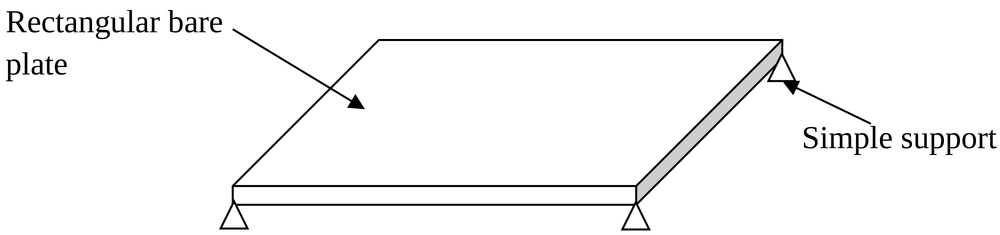


Figure 1. Simply supported rectangular bare plate.

For a simply supported plate the eigen function is:

$$\phi_{mn}(x) = \phi_m(x)\phi_n(y) \tag{1}$$

where ϕ is mode shape and *mn* are mode number.
The sinusoidal mode shapes in the *x* and *y* directions, respectively, are described by [24]:

$$\phi_m(x) = \sin(m\pi x / L_x) \ and \ \phi_n(y) = \sin(n\pi x / L_y) \tag{2}$$

where L_x and L_y are the length of plate in *x* and *y* directions, respectively.
The flexural displacement of a bare rectangular plate in modal space is given by [24]

$$w(x, y, t) = \sum_{mn} q_{mn}(t)\psi_{mn}(x) \tag{3}$$

where *q* is the modal coordinate and *m* and*n* are the mode numbers of the shape functions in the *x* and *y* directions, respectively, and

$$\psi_{mn}(x) = \psi_m(x)\psi_n(y) \tag{4}$$

are the mass-normalised eigen functions which satisfy the following orthogonality condition [25]:

$$\int_0^{L_x}\int_0^{L_y} \rho h \psi_{mn}\psi_{m'n'} dxdy = \{^{1 \quad mn=m'n'}_{0 \quad mn\neq m'n'} \tag{5}$$

$L_x$and L_y are respectively the lengths of the plate in the x and y directions, h is the plate thickness and ρ is the density.

For a plate simply supported on all four sides, the mass normalised eigen functions are given by:

$$\psi_{mn} = \frac{1}{M_n}\phi_{mn}(x) = \frac{1}{M_n}\sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi y}{L_y}\right) \tag{6}$$

where$M_n = \rho h L_x L_y/4$ is the modal mass.

Using the orthogonality condition, an expression for the kinetic energy of a bare plate becomes:

$$\begin{aligned} T &= \frac{\rho h}{2}\int_0^{L_x}\int_0^{L_y}\dot{w}^2(x)dxdy \\ &= \frac{\rho h}{2}\sum_{mn}\sum_{jk}\dot{q}_{mn}\dot{q}_{jk}\psi_{mn}(x)\psi_{jk}(x) \\ &= \frac{1}{2}\sum\dot{q}_{mn}{}^2 \end{aligned} \tag{7}$$

where$\dot{w}$denotes the derivative of w with respect to time.

Expression for the potential energy of the plate can be obtained as:

$$V = \frac{1}{2}\sum_{mn}\omega_{mn}^2 q_{mn}^2 \tag{8}$$

where $\omega_{mn} = \sqrt{\frac{D}{\rho h}}\left(\left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2\right)$

corresponds to the natural frequencies of the bare plate and

$$D = \frac{Eh^3}{12(1-v^2)}$$

is the plate flexural rigidity and E and v are respectively Young's modulus and Poisson's ratio. Lagrange's equation for a particular modal coordinate j is given by [24]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0, \quad j = 1,2......N \qquad [26]$$

Differentiating the kinetic and potential energies with respect to the modal coordinate and substituting into Lagrange's equation results in the equation of motion of the bare plate:

$$\ddot{q} + \omega^2 q = 0 \tag{10}$$

The natural frequencies can then be obtained by eigenvalue analysis [27].

Uncertain Mass Loaded Plate

Now consider the uncertain mass loaded plate as shown in Figure 2. For this the equation of motion is:

$$\ddot{q}_{pq} + \sum_{N_m}\sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m)\psi_{pq}(x_m) + \omega_{pq}^2 q_{pq} = 0 \qquad (.11)$$

where N_m is the number of point masses, $\psi_{mn}(x)$ are the mass-normalised eigen functions, and x_m corresponds to the random locations of the added masses.

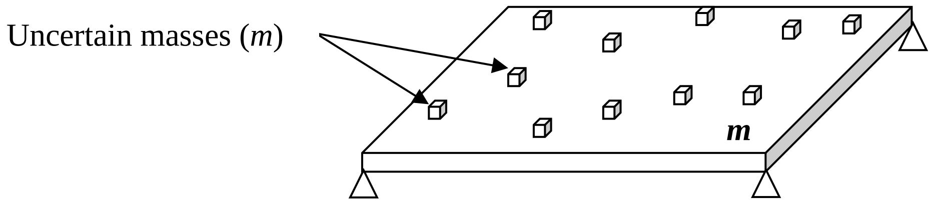


Figure 2. Simply supported rectangular plate with mass uncertainty.

Uncertain Spring (Stiffness) Loaded Plate

Now consider the uncertain spring loaded plate as shown in Figure 3. For this the equation of motion is:

$$\ddot{q}_{pq} + \sum_{N_m}\sum_{mn} k q_{mn} \psi_{mn}(x_k)\psi_{pq}(x_k) + \omega_{pq}^2 q_{pq} = 0 \qquad (.12)$$

where N_k springs with ground (of stiffness k) and $\psi_{mn}(x)$ are the mass-normalised eigen functions and X_k correspond to the random locations of the added springs

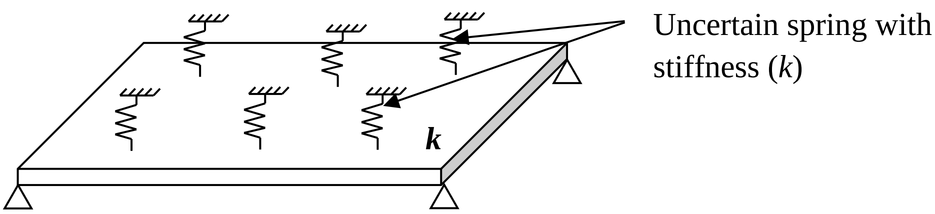


Figure 3. Simply supported rectangular plate with stiffness uncertainty.

Uncertain Mass-and-spring-loaded Plate

Now consider a mass-and-spring-loaded plate as shown in Figure 4. For this the equation of motion is [16]:

$$\ddot{q}_{pq} + \sum_{N_m}\sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m)\psi_{pq}(x_m) + \sum_{N_m}\sum_{mn} k q_{mn} \psi_{mn}(x_k)\psi_{pq}(x_k) + \omega_{pq}^2 q_{pq} = 0 \quad (13)$$

where N_m number of point masses and N_k springs to ground (of stiffness k) and $\psi_{mn}(x)$ are the mass-normalised eigen functions, and x_m and x_k respectively correspond to the random locations of the added masses and springs.

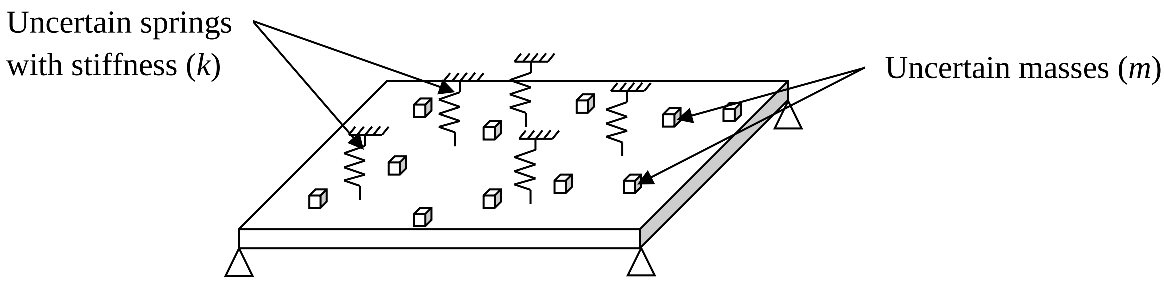


Figure 4. Simply supported rectangular plate with mass and stiffness uncertainty.

MODELLING OF RECTANGULAR PLATE WITH AND WITHOUT UNCERTAITY

A rectangular plate of dimension 500 mm × 600 mm × 2 mm of steel material with properties of $\rho = 7.86\times10^{-9}$ tone/mm^3,$\nu = 0.3$, $Y = 2\times10^5$ MPa was created. All material properties and simply-supported boundary conditions were applied to all edges. The type of element selected is quadratic shell element 181. Shell 181 is suitable for analysing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node translations in the *x*, *y*, and *z* directions, and rotations about the x-, y-, and z-axis.

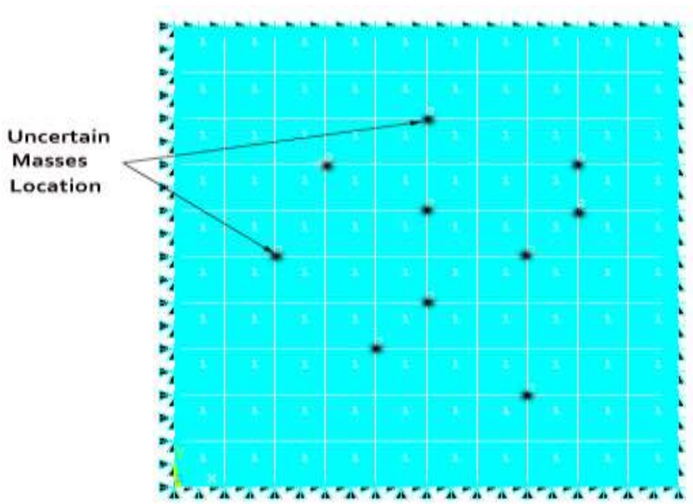


Figure 5. Discrete rectangular plate with 10 uncertain mass elements.

The degenerate triangular option should only be used as filler elements in mesh generation. Shell 181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. The change in shell thickness is accounted for in nonlinear analyses [28]. The total mass of the plate is 4.698×10^{-2} tons. Mass uncertainty is taken as 2% of the total mass [1]. The number of masses taken is 10. All are randomly placed on a plate and located by black spots in Figure 5[4]. The discrete rectangular plate with the location of 10 uncertain mass elements is shown in Figure 5. For uncertain masses 'mass 21 element' is taken which has three degree of freedom at each node [28].

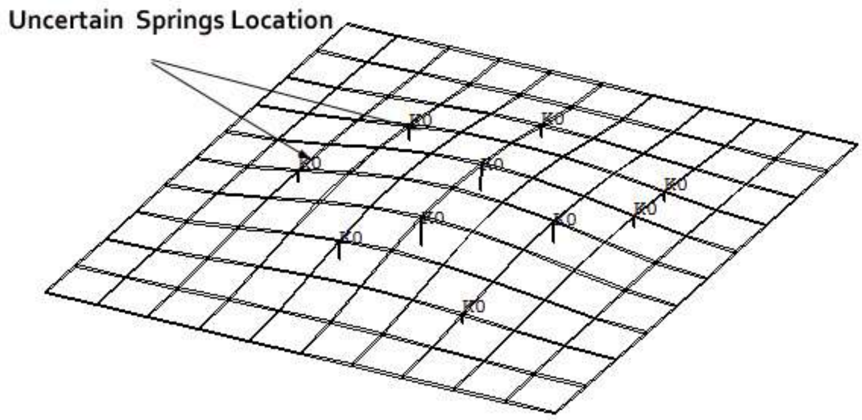


Figure 6. Discrete rectangular plate with 10 uncertain spring elements.

The 10 spring elements are added at the random locations. The locations are the same as the locations of the uncertain masses in Figure 6. The stiffness of each spring is 0.1N/mm. The stiffness element used is 'COMBIN14'. It has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node translation in the nodal *x*, *y*, and *z* directions [28].

EFFECT OF UNCERTAIN PARAMETERS ON NATURAL FREQUENCY AND MODE SHAPES

The frequency range is taken from 0 Hz to 500Hz. The plate is excitedin10 modes. The solver used is Block Lanczos [28]. It gives 10 natural frequency values for the bare plate, mass uncertainty plate, stiffness uncertainty plate and a combination of mass and stiffness uncertainty plates as shown in Table 1 and also in Figure 7 and Figure 8.

Table 1. Natural frequency value for 10 mode shapes.

Mode No.	Bare plate (Hz)	Mass uncertain plate (Hz)	% decrease in frequency due to mass uncertainty	Stiffness uncertain plate (Hz)	% increase in frequency due to stiffness	Mass and stiffness uncertain plate (Hz)	% increase in frequency due to mass and stiffness
1	32.481	32.413	0.21	60.39	85.92	60.29	85.62
2	72.53	72.41	0.166	110.12	51.82	109.9	51.52
3	90.16	90	0.178	136.369	51.25	136.094	50.95
4	130.4	130.229	0.131	186.857	43.29	186.546	43.06
5	139.479	139.272	0.149	190.913	36.87	190.575	36.63
6	186.546	186.157	0.209	253.338	35.8	252.647	35.43
7	198.207	197.443	0.387	273.848	38.16	273.358	37.92
8	227.385	227.076	0.136	301.768	32.71	301.227	32.47
9	234.237	233.9	0.144	309.504	32.13	308.881	31.87

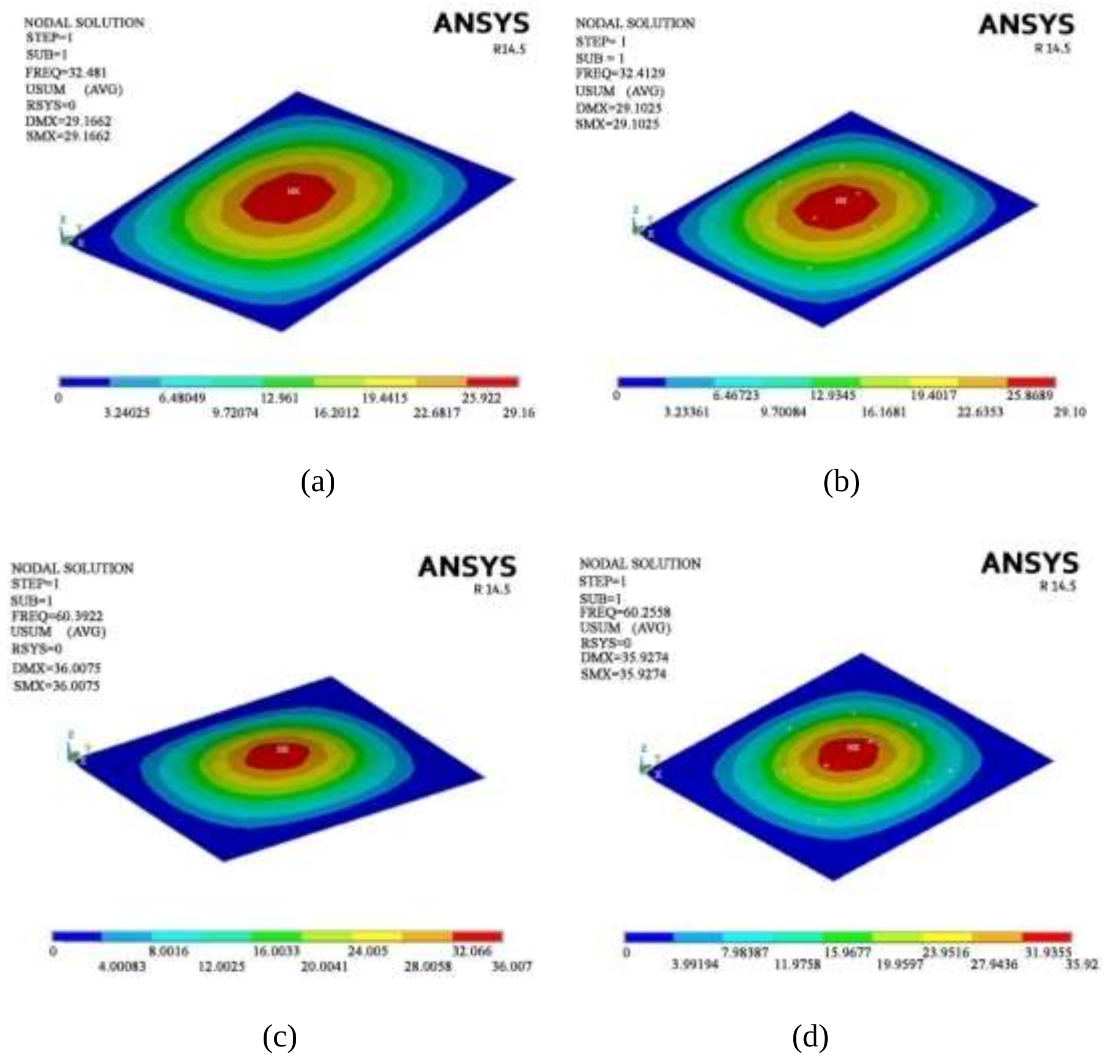


Figure 7. First mode shape of the plate: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty, (d) mass and stiffness uncertainty.

The natural frequency is reduced due to mass uncertainty and its percentage decreased is shown in Table 1. Due to stiffness uncertainty, natural frequency very prominently increases. These are the effects of uncertainty on the natural frequency. In the first mode, there is no change in mode shape obtained, as shown in Figure 7. However, in the second mode, as shown in Figure 8, due to mass uncertainty the mode shape is changed and due to stiffness the phase is changed. These effects of uncertainty on the mode shapes were found.

PREDICTING FREQUENCY RESPONSE FUNCTION

The frequency response function (FRF) is obtained by harmonic analysis [29]. Harmonic analysis of the plate is done for all edges simply supported. The same plate models are considered with all the uncertain parameters and of the same dimensions, which were considered in modal analysis. In this force vibration study, a force of 1 N is applied at node location 211, as shown in Figure 9. A frequency range of 0 Hz to 500Hz is given.

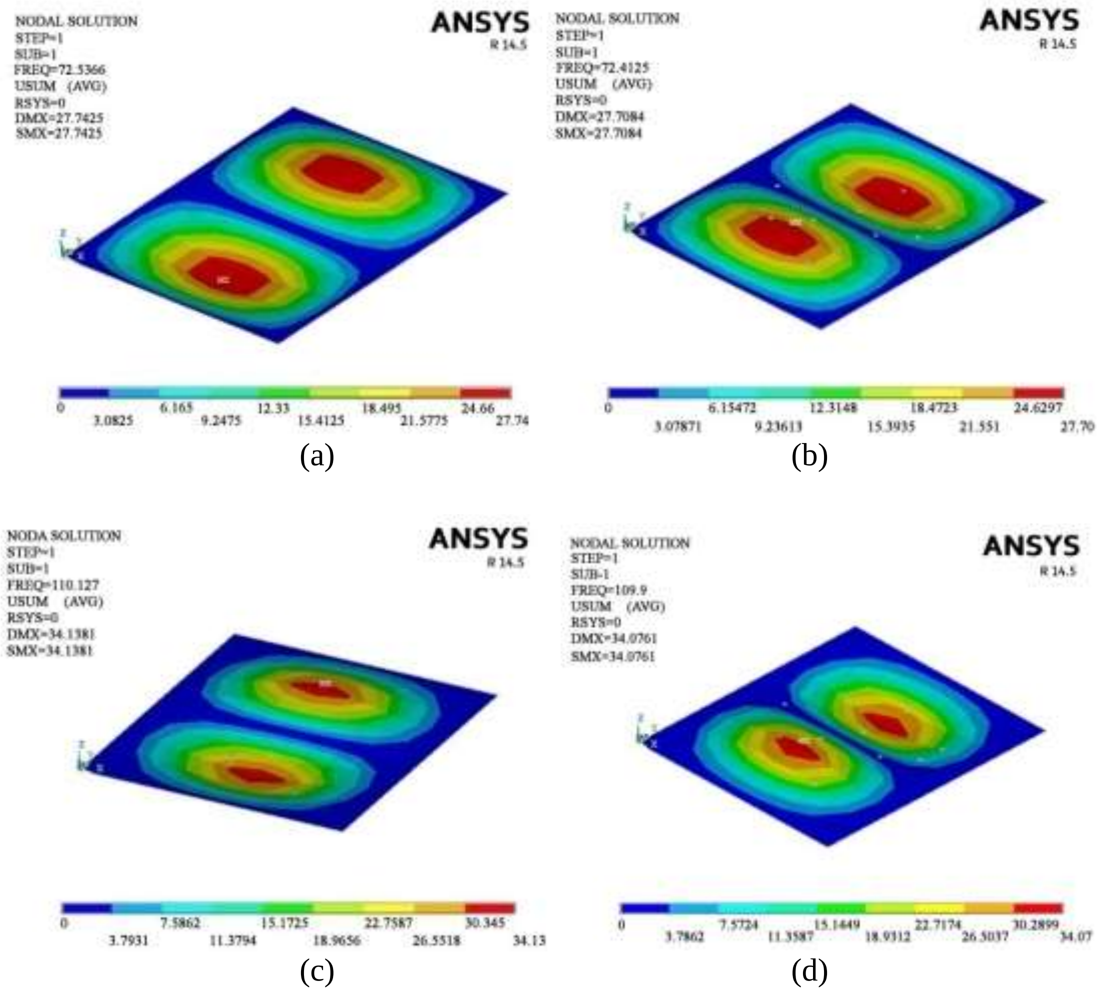


Figure 8. Second mode shape of the plate: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty, (d) mass and stiffness uncertainty.

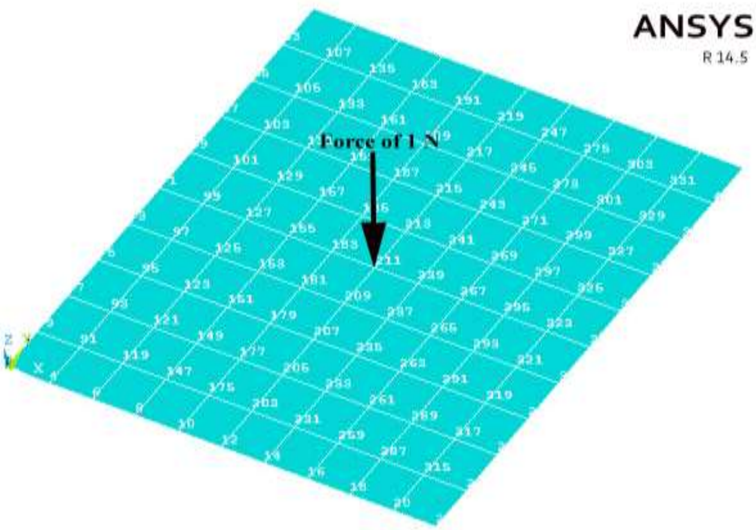
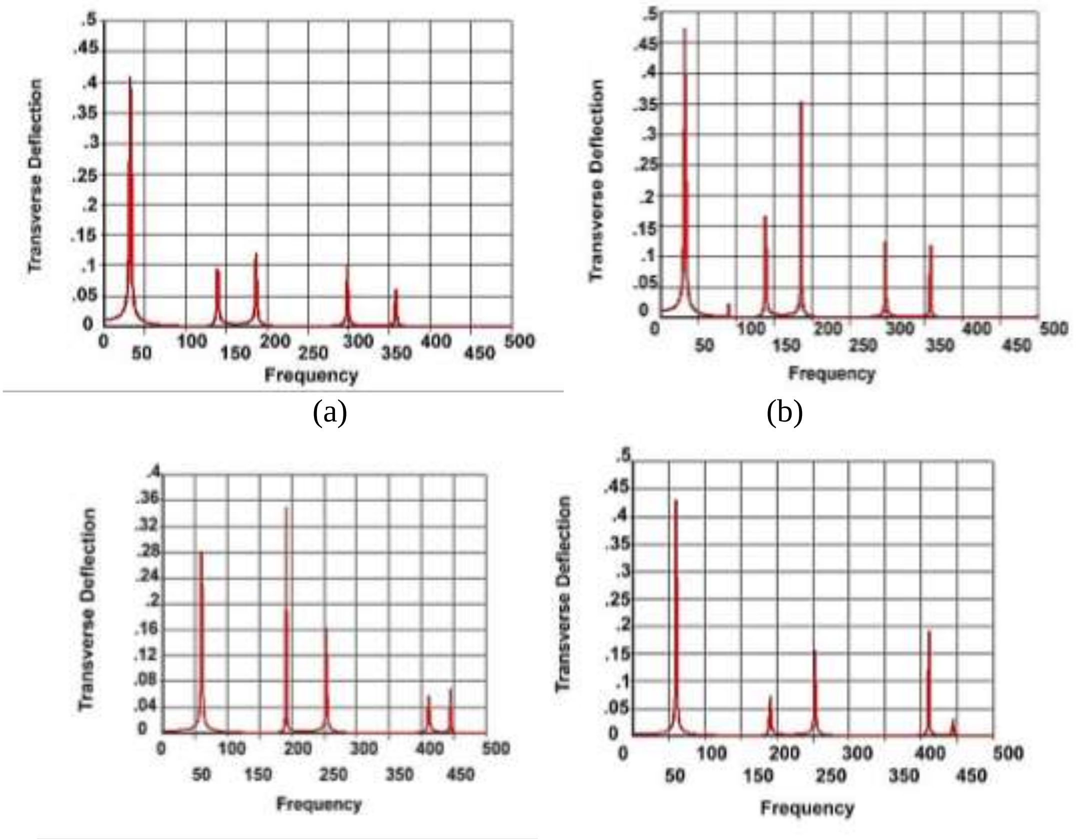


Figure 9. Force loading plate for harmonic analysis.

Frequency Response Function Analysis

An FRF was plotted on a linear graph scale. The frequency (Hz) is on the x-axis and amplitude (mm) on the y-axis, as shown in Figure 10. At node number 127of the plate FRF is taken. The FRF for the bare plate is shown in Figure 10(a) in which first resonance pick occurs at a frequency of 32.481Hz and amplitude of 0.41mm. In Figure 10(b) amplitude is increased by 0.47mm because of mass uncertainty at the same frequency. In stiffness uncertainty, Figure 10(c), major changes are observed which indicate that the complete resonance points are shifted to another frequency and their amplitude is also changed. For stiffness and mass uncertainty, Figure 10(d), the frequency of the resonance point is the same, but its amplitude is changed compared with the stiffness uncertainty.

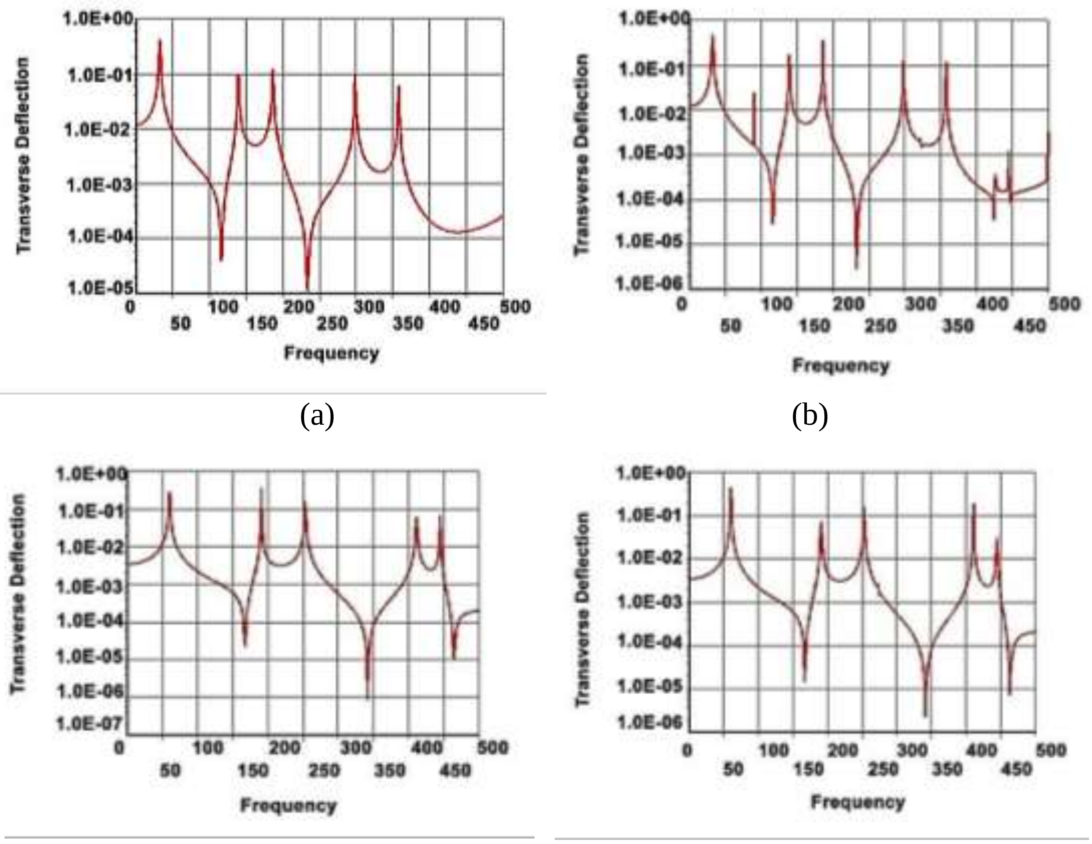
To know the response in more details it is necessary to plot it on a log scale, as shown in Figure 11. This figure is a harmonic response of the plate at node number127 in the log scale. Here it clearly shows node and antinodes points which are missing in the linear scale (Rao, 2000).Figure 11(a) shows the response of the bare plate which indicates its resonance and anti-resonance point of its different modes. Due to mass uncertainty at frequencies between400 Hz and500 Hz some different modes of the plate get excited, which was not at the bare plate, as shown in Figure 11(b). Similarly, in stiffness and mass and stiffness uncertainty resonance peaks are shifted to another frequency, the magnitude of the amplitude is also changed, as shown in Figure 11(c, d), compared with the bare plate.



(a) (b)

(c) (d)

Figure10. FRF Plot on Linear Scale: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty, (d) mass and stiffness uncertainty.



(a) (b)

(c) (d)

Figure 11. FRF Plot on Log Scale: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty, (d) mass and stiffness uncertainty.

RESULTS AND DISCUSSION

During the modal analysis of the plate, as shown in Table 1, in the fundamental mode the natural frequency of the plate with mass uncertainty decreased by 0.21% compared with the bare plate. Due to stiffness uncertainty the natural frequency is drastically increased by 85.92%. Due to mass and stiffness uncertainty the natural frequency is increased by 85.62%. In the harmonic analysis the FRFs are plotted on a linear as well as on a log-log scale. All the FRFs of node 127 for all the parameters of the plate are combined together in Figures 12 and 13 to study the characteristics of the plate effectively. All edges in the simply supported condition plate are excited by an external force and the response of the plate is plotted on the FRF graph, as shown in Figure 12. The frequency of the bare plate in the first fundamental mode (green colour) is32 Hz and the amplitude is 0.408448 mm. The mass uncertainty (black colour)is added and the frequency obtained is 32 Hz and the amplitude is 0.474298mm. When the stiffness (blue colour) uncertainty is added a very prominent pick with a major shift in frequency of 60Hz and large amplitude of 0.281087mm is obtained. Due to mass and stiffness uncertainty (red colour)a major frequency shift of 60 Hz and amplitude of 0.429595mm is obtained. The complete resonance point is shifted.

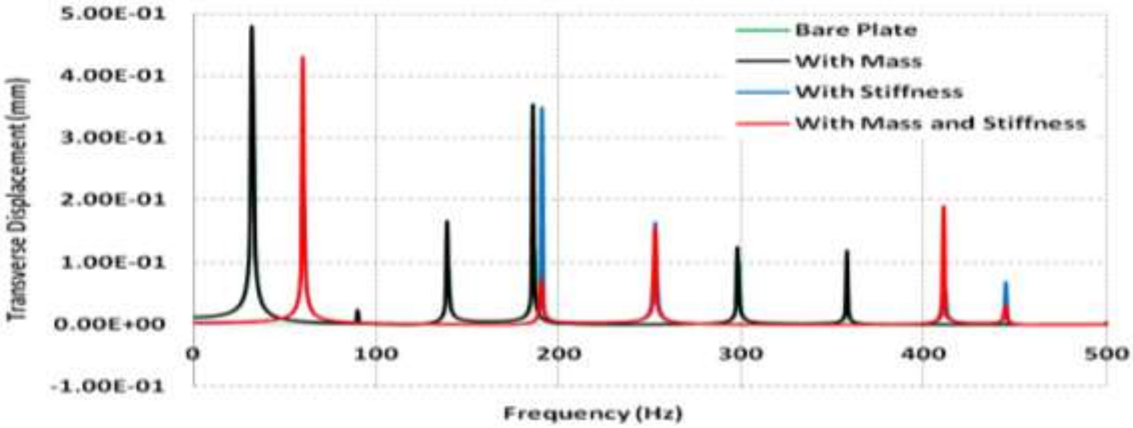


Figure 12. Comparison Linear Plot of Node 127.

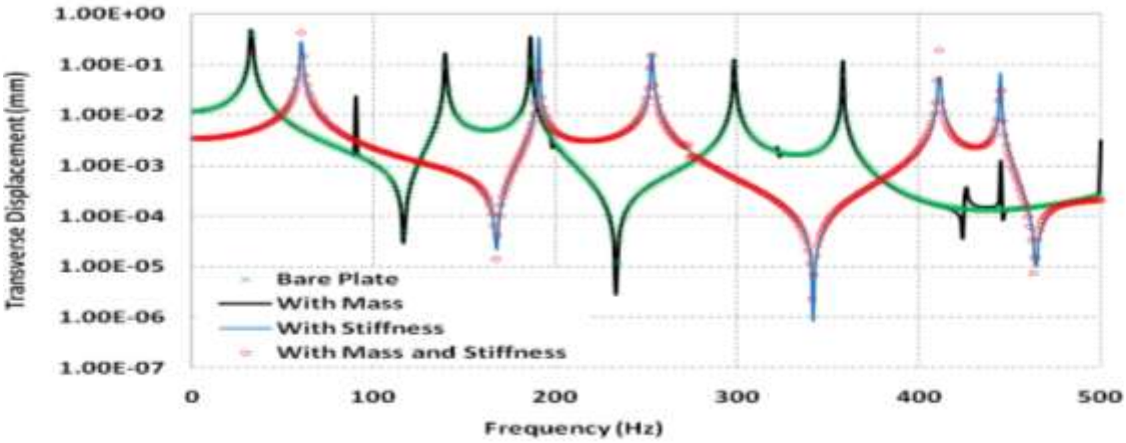


Figure 13. Comparison Log Plot of Node 127.

The FRF of the plate for all edges, simply supported condition is shown in Figure 13. The response of the bare plate is compared with the all other uncertain parameters. In this log-log scale, many details of plate characteristics are studied. In the fundamental mode due to stiffness (blue colour) and a combination of mass and stiffness (red colour) uncertainty, the resonance point is completely shifted compared with the bare plate (green colour). Due to mass uncertainty (black colour) at a high frequency between 50Hz and100Hz and 400 Hz and450Hz different modes of vibration are obtained which were not present in the bare plate characteristic. The completely different characteristics of the plate are obtained due to mass uncertainty.

CONCLUSIONS

In this analysis four different mass and stiffness uncertain parameters were considered and their effects on vibrational characteristics of the plate were found. During analysis, it was found that due to mass uncertainty very minor changes occurred in the value of natural frequencies. However, due to stiffness uncertainty, it changed dominantly. Due to mass uncertainty mode shape changed and due to stiffness uncertainty its phase was changed. At a high-frequency domain some hidden mode was excited due to a mass uncertainty which was not previously known in the bare plate response. Uncertainty affects resonance points due to stiffness uncertainty changes which occur drastically, the complete picks of resonance were shifted and amplitude also changed. Due to mass uncertainty much less change occurred at the resonance points. Because of uncertainty the complete vibrational characteristics were changed. Therefore it becomes important to consider these uncertainties otherwise misinterpretation will occur while designing plates.

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